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Measurement of the Relativistic Doppler Effect Using 8.6-MeV Capture γ Rays

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The energies of γ rays from the 10.27-MeV ($T = 1$, $\Gamma < 1.8$ keV) $\rightarrow$ 1.63-MeV ($T = 0$) transition in ^{20}Ne have been measured in an annular Ge (Li) detector at 0 and 180°. $^{16}O(^4He,\gamma)$ and $^4He(^{16}O,\gamma)$ capture reactions in a differentially pumped gas target were used to produce recoiling ^{20}Ne ions with two different very accurately known velocities. We observe a tranverse Doppler shift of 10.09 ± 0.41 keV for the ^{16}O capture reaction. This allows us to determine relativistic time-dilation effects at velocities of $0.05c$ to an accuracy of 3.5%. Our results are in very good agreement with the prediction of special relativity.

I. INTRODUCTION

A crucial test of the theory of special relativity was the measurement of the transverse Doppler shift by Ives and Stilwell in 1938.[1] The relativistic Doppler shift formula,

$$\nu_{\text{lab}} = \nu_0 (1-\beta^2)^{1/2} / (1-\beta \cos\theta_{\text{lab}}), \qquad (1)$$

may be derived directly from the Lorentz transformation.[2] This phenomenon is a geometrical property of space-time, and is intimately connected with the problem of synchronization of clocks in different frames of reference. Several authors [3-5] have critized Einstein's treatment of this problem.

Equation (1) is often expanded as a power series in β, omitting terms of order β^3 and higher, and with $E = h\nu$:

$$E_{\text{lab}} = E_0 + E_0(\beta\cos\theta_{\text{lab}} + \beta^2\cos\theta_{\text{lab}}) - \tfrac{1}{2}E_0\beta^2,$$

where the second and third terms are called the longitudinal and transverse shifts, respectively.

In principle the transverse shift may be measured directly at $\theta_{\text{lab}} = 90°$ when the instrumental resolution is sufficient. However, at 90° the derivative of the longitudinal shift is a maximum, so that the longitudinal shift for $\theta_{\text{lab}} = 91°$ is of the same order as the transverse shift, and the finite solid angle of the detector will cause a large broadening of the lineshape at 90°.

These difficulties may be circumvented by measuring near 0 and 180°, where the longitudinal shift changes slowly with angle. In this case the longitudinal shift is also measured, allowing a determination of β.

Ives and Stilwell observed Doppler-shifted optical photons emitted at 0 and 180° by a beam of excited hydrogen atoms. A similar experiment with higher-velocity beam ($\beta = 0.005 \rightarrow 0.009$) and improved accuracy has been performed in 1962 by Mandelberg and Witten[6] confirming the prediction of special relativity to an accuracy of 5% of the transverse Doppler shift. The work described in this paper is similar in concept to the Ives and Stilwell experiment, insofar as we measure the Doppler effect at 0 and 180° to test for terms of order β^2 and higher. We use high-energy γ rays of 8.64 MeV instead of optical transitions, and a recoil velocity of $\beta \simeq 0.05$.

The present experiment measures accurately at 0 and 180° the energies of Doppler-shifted γ

rays produced in a capture reaction. In the reaction chosen, $^{16}O + {}^{4}He \rightarrow {}^{20}Ne^*$, we populate the 2^+, $T=1$ state at 10.27 MeV in ^{20}Ne, and study the shift of the 8.64-MeV γ ray emitted in the transition from this state to the 2^+, $T=0$ state at 1.63 MeV. The width of the level at 10.27 MeV is less than 1.8 keV.[7] The capture reaction produces a well-collimated beam of ^{20}Ne recoils with a very sharp velocity distribution. Measurements were made of γ rays from the resonance for two recoil velocities. When a ^4He target was bombarded by 27.7-MeV ^{16}O ions, the recoil velocity was $0.049c$. When an ^{16}O target was bombarded by 6.93-MeV ^4He ions giving the same center-of-mass energy, the recoil velocity was $0.012c$. The high recoil velocity produced a transverse Doppler shift of 10 keV, which could be accurately measured with a Ge (Li) detector of resolution 7.2 keV (FWHM) at 8.6 MeV. By measuring the γ-ray energies at two different recoil velocities we were able to avoid the problem of determining an absolute energy calibration; thus a major source of systematic error was eliminated.

II. APPARATUS

A schematic diagram of the experimental apparatus is shown in Fig. 1. A highly collimated beam from the Chalk River MP Tandem accelerator passes into a target chamber before the annular Ge (Li) detector, then down a beam tube through the center of the detector and into a second target chamber followed by a beam catcher. The target chambers were differentially pumped gas cells, and the front and back chambers, for measurements at 0 and 180°, respectively, were pressurized to 2 Torr.

The location of the reaction in the gas cell was determined by measuring the yield as a function of beam energy of high-energy γ rays (~6–10 MeV) with a 12.7-cm diam x 15.4-cm long NaI detector

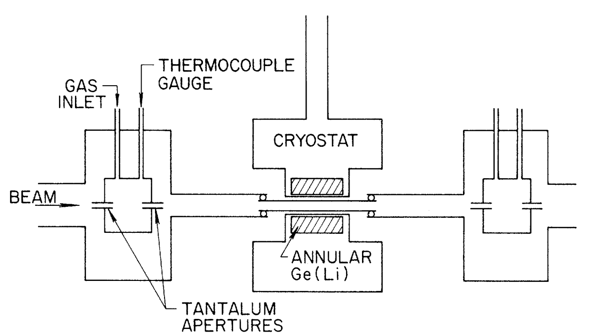

FIG. 1. Schematic arrangement of detector and gas cells.

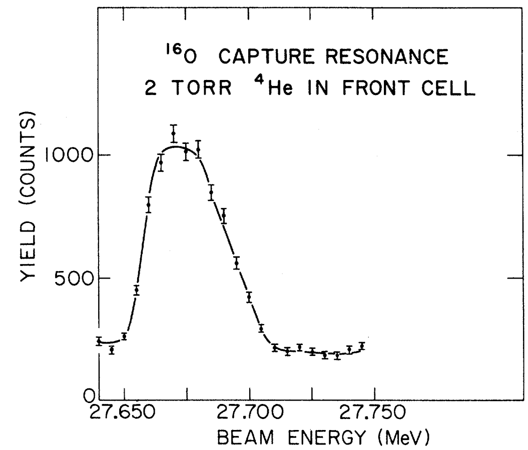

FIG. 2. Oxygen-capture resonance in the front cell, measured with a NaI(Tl) detector.

placed close to the cell. A typical yield curve for 2 Torr ^4He in the front cell is shown in Fig. 2. The ^4He pressure is sufficient to produce a flat-topped thick target yield curve. The front-edge slope of the curve results from a combination of the resonance width and the beam energy spread of 0.05%. The longer tail on the high-energy side is due to additional beam energy straggling and to reactions in the residual gas in the Ge (Li) detector through tube. The latter assumption is supported by the observation of a similar tail at the low-energy side when the back cell is pressurized. During the experiment the position in the cell for which the beam energy was on resonance was located at front edge of the front cell by selecting the lowest energy for full yield, and at the back edge of the back cell by choosing the highest energy for full yield.

The highest practicable pressure was chosen to obtain the best spatial resolution and high yield. The high-pressure limit arises from residual gas in the Ge (Li) detector through tube. The detector subtends a high solid angle for reactions occurring in this gas and hence has a much higher detection efficiency than for those taking place in the gas cell. A series of different partial pressures was studied and 2 Torr chosen as the optimum.

The distance from the center of each gas cell to the center of the Ge (Li) detector was 17.8 cm. The geometry of the detector was accurately known and the position in the 4.5-cm cell where the resonance reaction was occurring could be located to better than ± 1.5 cm. The average angle (with respect to the recoil direction) of the γ rays absorbed in the detector was determined from a geometrical calculation to be $|\langle\cos\theta\rangle| = 0.9963 \pm 0.0010$. The principal source of error in calculating $\langle\cos\theta\rangle$ is the uncertainty in locating the resonance position in the gas cell.

The annular Ge (Li) detector[8] had an active volume of 44 cm^3 and a resolution (FWHM) at 3.85 MeV of 6.1 keV. The diameter of the tube through the center of the detector cryostat was 9 mm. The detector cryostat was mounted so that it could be rotated 180° about an axis perpendicular to the beam during the course of the experiment in order to compensate for possible inhomogeneities in the detector.

III. EXPERIMENTAL MEASUREMENTS

The experimental measurements were performed in the following order:

The front and rear cells were alternately filled to 2 Torr with ^{4}He gas and the γ-ray spectrum recorded during bombardment with ^{16}O ions. This was repeated several times. The sums of the spectra obtained at 0 and at 180° are shown in the lower half of Fig. 3.

The cells were then alternately filled to 2 Torr with ^{16}O gas and bombarded with ^{4}He ions. This was also repeated several times and the sums of these spectra are shown in the top half of Fig. 3. Because the recoil velocity in the second case is only $\frac{1}{4}$ that in the first, the separation between the 0 and 180° peaks is much smaller. The Ge (Li) detector was then rotated through 180° and the series of runs repeated. Within the statistical error there was no difference in the observed separations.

During the runs, two mercury relay pulse generators were fed into the preamplifier, one giving a peak at the low end of the spectrum and the other at the top end. The stability of the reference voltage supply for the top end pulser was monitored and recorded on a strip recorder throughout the experiment. Fluctuations in this voltage were found to be less than one part in 10^5. The peaks from these pulsers were used in a computer-controlled stabilization system acting

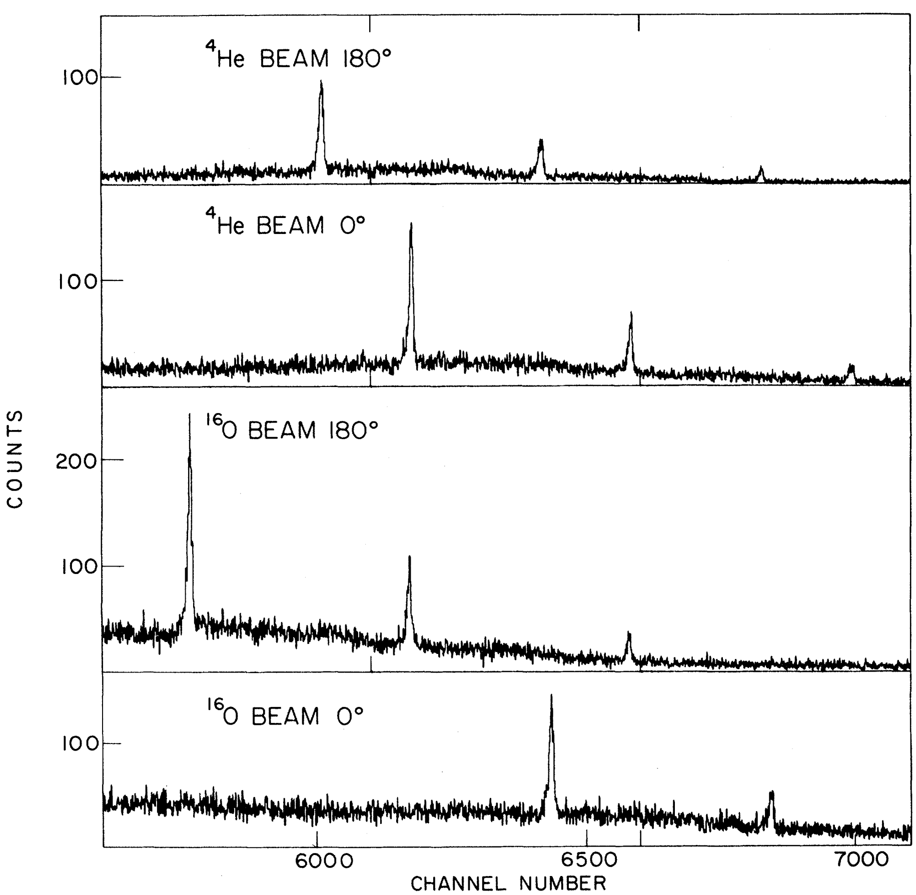

FIG. 3. γ rays observed in the Ge(Li) detector. The 0–180° Doppler shift in the lower spectra is 840 keV, and in the upper spectra, 210 keV.

on both the zero and gain of the analog-to-digital converter.

During the experiment γ rays from a RdTh source were seen by the Ge (Li) detector. The positions of the 2.614-MeV peak and its 1.592-MeV double escape peak were determined in each run. This checked the stability of the analyzing system. Small shifts were observed and corrections were made to the data as discussed later.

IV. RESULTS

The energies of the double and single escape peaks, given in Table I, were determined by maximizing their overlap with a Gaussian peak shape whose width was set to the resolution of the Ge (Li) detector. This procedure was used to minimize the contribution of the low-energy tail to the peak. More sophisticated fitting techniques were also applied to the data, but we found that the poor statistics on each individual run led to large statistical variations in the width and other parameters fitted. For each 2-hour run a separate fitting was done, and the energy was calculated relative to the ThD lines at 2614.7 keV and 1592.4 keV. The peak energies so obtained were then averaged to obtain the results of Table I. Although the centroids of the ThD lines were determined to better than 0.07 keV for each run, the resultant errors in the determination of gain and zero contributed significantly to the error in our final result because of the extrapolation involved. Drifts in the ThD centroids for adjacent runs were less than 0.1 keV; this however amounted to 1.8 keV over the entire experiment.

The errors quoted in Table I are calculated from counting statistics. Account was taken of the nonlinearities in the electronics for the calculation of the energies, but not in the calculation of the errors. The discussion of systematic errors of this sort will be deferred to Sec. V. The unnormalized χ^2 values given in Table I show that our results are statistically consistent among individual runs. Furthermore the counting statistics are so much better for double escape peaks than single escape peaks that the latter will have little effect on our final values.

Analysis of the Data

In order to test relativity with our data we must extract the transverse Doppler shift and then look for residual relativistic effects. To achieve this we assume that the data may be described by the equation

$$E(\theta)=\frac{E_0F(\beta)}{1-\beta\cos\theta}, \tag{2}$$

where θ is the angle between the recoil velocity β and the direction of propagation of the γ ray, measured in the laboratory frame. In special relativity $F(\beta)$ is $(1-\beta^2)^{1/2}$. Equation (2) holds for general relativity, where F is also a function of the gravitational potential, and for classical physics, where $F=1$.

From Eq. (2) we derive

$$\beta\langle\cos\theta\rangle=\frac{\overline{E}(0°)-\overline{E}(180°)}{\overline{E}(0°)+\overline{E}(180°)}. \tag{3}$$

Here $\overline{E}$ is the mean energy of the γ rays absorbed in the counter, and $\cos\theta$ is averaged over the counter solid angle. $\langle\cos\theta\rangle$ is known from the geometry of our gas cell to be 0.9963 ± 0.0010, assuming the resonance is contained within ±1.5 cm in the gas cell; this assumption seems well justified by the shape of the resonance curves. In Table II we compare these values of β [calculated from Eq. (3) and thus nonrelativistic, independent of $F(\beta)$] with values of β calculated using relativistic particle kinematics, for which the input data are the masses of ^{16}O, ^{4}He, ^{20}Ne and the energy of the resonance capture level in ^{20}Ne. The uncertainty of 2 keV in the latter energy determines the errors assigned to this calculation.

The velocity determined from Eq. (3) will be a velocity relative to the ether,[2] while the particle kinematics calculation will yield a velocity with

TABLE I. Peak energies of Doppler-shifted ^{20}Ne γ rays.

		Double escape				Single escape				Separation	
Beam	Angle	Energy (keV)	Error (keV)	χ^2	No. of runs	Energy (keV)	Error (keV)	χ^2	No. of runs	Energy (keV)	Error (keV)
^{16}O	0	8045.27	0.32	8.83	7	8556.69	0.58	2.44	7	511.35	0.63
^{16}O	180°	7205.50	0.27	5.78	5	7716.37	0.45	1.78	5	510.86	0.48
^{4}He	0	7721.08	0.32	1.64	6	8231.35	0.48	2.57	6	510.30	0.47
^{4}He	180°	7510.36	0.37	1.15	3	8020.96	0.54	3.28	3	510.58	0.54

TABLE II. Recoil velocity of ^{20}Ne determined from the Doppler shift and from kinematics.

	v/c(^{16}O beam)	v/c(^{4}He beam)	v(^{16}O)/v(^{4}He)
Doppler shift	0.048 736(0.000 024)	0.012 242(0.000 029)	3.9810(0.0096)
Kinematics	0.048 721(0.000 009)	0.012 192(0.000 003)	3.9961(0.0000)

respect to the laboratory. Thus a comparison of these values is really a test of the ether-drift velocity, for which we obtain an upper limit of 15 km/sec. Much lower limits have been set for this velocity in other experiments.[9,10] Our experiment is not really sensitive to ether drift because our recoil velocity is so much greater than the ether drift limits. A convenient test of relativity can be made through the ratio

$$R \equiv \frac{F(\beta)^{(^{16}\mathrm{O\ beam})}}{F(\beta)^{(^{4}\mathrm{He\ beam})}} = \frac{\left(\dfrac{1}{\overline{E}(0°)}+\dfrac{1}{\overline{E}(180°)}\right)^{^{4}\mathrm{He\ beam}}}{\left(\dfrac{1}{\overline{E}(0°)}+\dfrac{1}{\overline{E}(180°)}\right)^{^{16}\mathrm{O\ beam}}}. \tag{4}$$

Using our measured peak energies from Table I, we obtain, from Eq. (4),

$$R = 1 - 0.001\,093 \pm 0.000\,038.$$

With β determined nonrelativistically by Eq. (3) and $F(\beta) = (1-\beta^2)^{1/2}$ from special relativity we find $R = 1 - 0.001\,114$. Thus our result agrees with the prediction of special relativity. If we parametrize $F(\beta) = (1-\beta^2)^A$ (which goes to zero as $\beta \rightarrow 1$) then $A = 0.491 \pm 0.017$. Mandelberg and Witten[6] obtain $A = 0.495 \pm 0.025$. Note that formulas (3) and (4) are independent of an absolute energy scale since they involve only energy ratios. The only significant errors in these quantities are in the determination of gain and zero shifts between runs and the statistical uncertainties in the positions of the γ-ray lines.

We may also analyze the experiment in a less general manner that however has the advantage of expressing our result more directly in terms of the measured quantities. Using the energies from the $\beta = 0.012$ recoils and assuming $F(\beta) = (1-\beta^2)^{1/2}$ at this recoil velocity, we may calculate E_0 from Eq. (2). The quantity $E_0 F(\beta)$ for the $\beta = 0.049$ recoils is calculated from

$$E_0 F(\beta) = 2[E(0°)^{-1} + E(180°)^{-1}]^{-1}. \tag{5}$$

Then the relativistic shift of the γ rays observed at $\beta = 0.0487$ is $E_0[1 - F(\beta)] = 10.09 \pm 0.41$ keV compared with a shift of 10.26 keV predicted by $F(\beta) = (1-\beta^2)^{1/2}$. In time dilation measurements the relativistic correction is usually expressed as $\gamma - 1$, where $\gamma = 1/F(\beta)$. Our result may be expressed as $\gamma - 1 = 0.001\,165 \pm 0.000\,040$ at $\beta = 0.0487$.

Systematic Error and Consistency

The chief source of systematic error in this experiment is nonlinearities in the detector system. An integral nonlinearity of 0.1% over 800 channels would produce a systematic error in R equal to the quoted statistical error. Using a precision mercury relay pulser and a precision voltage source we measured the integral nonlinearity of our electronic equipment and obtained a correction curve. Over the region of interest, channels 5500 to 6800, we obtained an integral nonlinearity of 0.022% in this manner. A similar determination of the integral nonlinearity was made using a sliding ramp pulser; however the specified linearity of this pulser was only 0.1%. Using these two different correction curves to evaluate the exponent we obtained $A = 0.494$ from the sliding ramp pulser compared with $A = 0.491$ from the mercury relay pulser. We estimate that after the linearity correction from the mercury relay pulser is made, residual nonlinearities will be about 0.01%. This is insignificant compared with the statistical error.

To eliminate the possibility of nonlinearities in the germanium detector itself, the linearity was also measured using well-known γ-ray peaks from (n, γ) reactions in Fe and Ni. Each γ ray gives peaks at E_γ, $E_\gamma - m_0c^2$, and $E_\gamma - 2m_0c^2$. From measurements of these separations we have determined that the linearity over 1 MeV is better than 0.05%. The possibility of nonlinearities in the Ge (Li) detector arises from a report of deviation from m_0c^2 in the separation of single and double escape peaks at high γ energy as measured with a planar Ge (Li) detector[11] when the γ ray is incident in the direction of the applied bias electric field. No such effect is expected or has been seen in our experimental configuration where the γ ray is incident perpendicular to the bias field.

Our value for the ratio R is not affected by the geometrical uncertainties. From the expansion $F(\beta) \cong 1 - A(\beta\langle\cos\theta\rangle/\langle\cos\theta\rangle)^2$, where $\beta\langle\cos\theta\rangle$ is determined from (3), and $\langle\cos\theta\rangle$ from a geometrical calculation, it is clear that the uncertainty in $\langle\cos\theta\rangle$ of

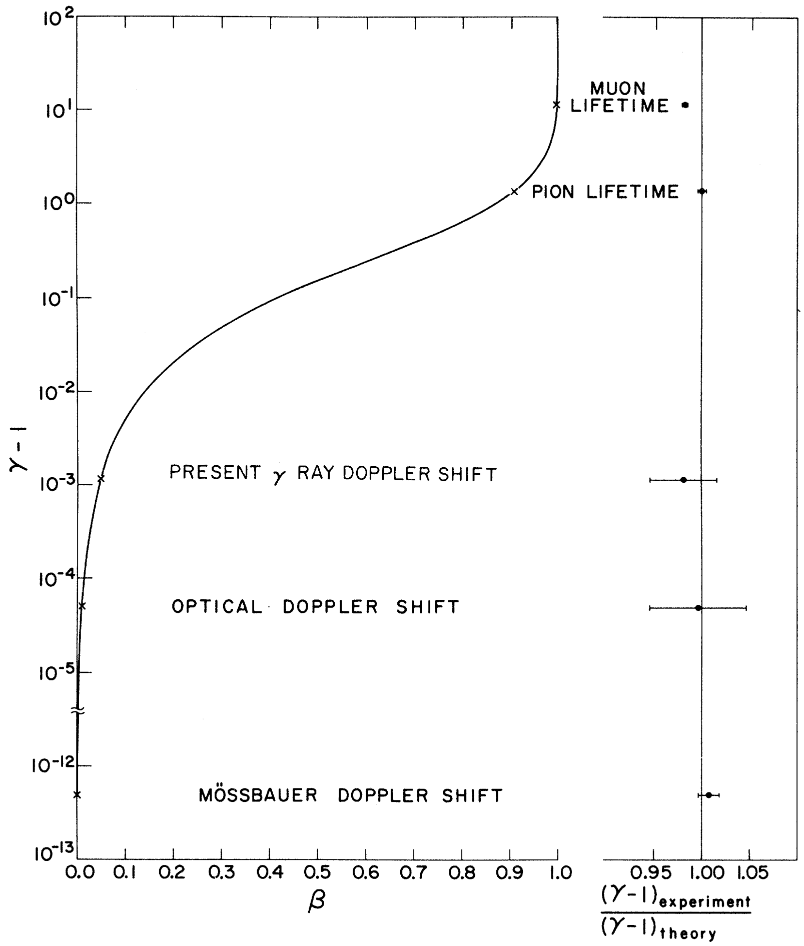

FIG. 4. Summary of the more accurate Doppler-shift and time-dilation measurements.

0.1% is negligible compared with the 3.5% statistical error in A.

Errors in the geometry and nonlinearities in the measuring system would each affect the agreement (see Table II) between velocities measured from the Doppler shifts and those calculated from relativistic kinematics. Since the ratio of the recoil velocities does not depend in either case on the form of $F(\beta)$, the agreement of this ratio provides a check on systematic errors that is independent of any relativistic assumptions.

V. DISCUSSION

Lorentz invariance is a very powerful postulate determining the structure of modern physical theory. In view of the far-reaching consequences of this postulate, it is important to investigate its validity under varied circumstances and with the highest possible experimental precision. In order to discuss the value of the present experiment in this perspective we will examine briefly some of the other experiments supporting special relativity.

(a) Experiment testing constancy of c. The Michelson-Morley experiment, as performed with microwave interferometers, has established that the value of c is independent of direction in space to very high accuracy.[12] In a direct measurement of the velocity of γ rays from the decay of relativistic neutral pions, the γ-ray velocity was found equal to c to an accuracy of 130 ppm.[13] Thus the experimental evidence for this postulate is strong. However, null experiments of this sort are not sensitive to the detailed form of the Lorentz transformation.

(b) Mass energy equivalence. The Einstein relation $E = Mc^2$ has been checked through comparison of reaction Q values with mass differences related through mass spectroscopy to the proton mass. The latter is then determined accurately from the proton cyclotron frequency. Agreement with the Einstein relation to about 35 ppm was obtained.[14] These authors point out that this analysis ignored the use of common calibrations for

several of the reactions (some of which were calculated with the help of the mass-energy relation), so that this error will increase when a more careful analysis is done. Nevertheless this result is a strong confirmation of Einstein's prediction. A related experiment is the measurement of the mass variation of 385- and 660-MeV protons, which agrees with theory to an accuracy of 0.2%.[15] These experiments test the dynamics of the theory, for we must assume, in addition to Lorentz invariance of space-time, that particles in an electromagnetic field are described by a scalar Hamiltonian, so that the 4-momentum p^μ is conserved.

(c) Time-dilation measurements. The relativistic contribution to the Doppler shift is due to time dilation. As shown in Fig. 4, a number of such measurements have been reported, spanning a wide velocity range. The lifetime of the μ^- meson was measured to 0.1% at $\beta=0.996$ as a spin-off of the $g-2$ experiment.[16] The 2% disagreement with the prediction of relativity is ascribed to meson losses in the storage ring.

The charged pion lifetime has been measured at $\beta=0.91$, confirming the value of $\gamma-1$ predicted by relativity to an accuracy of 0.4%.[17] In addition to the present measurement at $\beta=0.05$ (3.5% accuracy), relativistic Doppler shifts have been measured in neutral H beams at $\beta=0.01$ (5% accuracy),[1,6] and in ^{57}Fe at $\beta=10^{-6}$ using the Mössbauer effect (1.1% accuracy) (Ref. 10).

Although time dilation itself does not depend on dynamics, a breakdown of Lorentz invariance in the dynamics may lead to an additional anomalous Doppler shift. For example, a velocity dependence of the energy levels in ^{20}Ne because of a breakdown of Lorentz invariance in the strong interaction would cause such additional shifts. In the meson lifetime experiments the weak interaction is tested, while the atomic beam experiment tests electromagnetism.

Among the various Doppler-shift measurements, ours uses the highest velocity, and the Doppler-shifted radiation has the highest frequency. The simplicity of the kinematics in capture reactions allows us to test for possible systematic error. All the relevant data are obtained with the same apparatus, and no absolute calibrations are required.

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