RELATIVITY AND CLOCKS

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Summary

In this centennial year of the birth of Albert Einstein, it is fitting to review the revolutionary and fundamental insights about time which he gave us in the Restricted Theory of Relativity (1905) and in the conseqences of the Principle of Equivalence ("...The happiest thought of my life...") which he developed (1907-1915) into his theory of gravity as curved space-time, the General Theory of Relativity.

It is of particular significance that the extraordinary stability of modern atomic clocks has recently allowed the experimental study and accurate measurement of these basic effects of motion and gravitational potential on time. Experiments with aircraft flights and laser pulse remote time comparison (Alley, Cutler, Reisse, Williams, et al, 1975) and an experiment with a rocket probe (Vessot, Levine, et al, 1976) are briefly described.

Proper understanding and allowance for these remarkable effects is now necessary for accurate global time synchronization using ultrastable clocks, transported by aircraft, and for the correct operation of navigational systems such as the NAVSTAR/Global Positioning System.

Introduction

It is an honor to be asked to review the subject of relativity and clocks for the 33rd Frequency Control Symposium in this centennial year of the birth of Albert Einstein which occurred in Ulm, Germany on March 14, 1879. I am pleased to attempt a brief summary of the subject.

The plan is to recall first some of the significant events in Einstein's intellectual life by showing his photographs at various ages. Next, the restricted ("special") theory of relativity (restricted, that is, to inertial frames of reference) will be sketched, followed by the implications of the Principle of Equivalence that gravity curves light beams and affects the rate of clocks, the clues that led to the idea of gravity as curved space-time, the General Theory of Relativity. Recent experiments which have been able to measure and study some of the effects of motion and gravity on time will be briefly described. Finally, the practical matter of including relativity in modern clock synchronization and navigational systems will be mentioned.

The international timekeeping community should take great pride in the fact that the great stability of contemporary atomic clocks <u>requires</u> the <u>first practical applications of Einstein's General Theory of Relativity</u>. This circumstance can be expected to produce a better understanding among physicists and engineers of the physical basis of gravity as curved space-time. For slow motions and weak gravitational fields, such as we normally experience on the earth, the <u>primary curvature</u> is that of <u>time</u>, not space. A body falls, according to Einstein's view, not because of the Newtonian force pulling it to the earth, but because of the properties of time: clocks run slower when moving and run faster or slower, the higher or lower respectively they are in the earth's gravity field.

Some Events in Einstein's Intellectual Development

<u>Figure 1</u> shows Einstein in his study in Berlin several years after he had brought the General Theory of Relativity to its complete form in 1915. He always regarded this theory as his major accomplishment, although his other outstanding contributions to physics would place him among the greatest physicists of this century even without the General Theory. His stature as a scientist and his character as a man are appropriately symbolized in the cartoon of Herblock which appeared in The Washington Post shortly after his death on April 18, 1955. (<u>Figure 2</u>). This drawing is included here for an additional reason.

Imagine someone observing the Earth from the position of Herblock's observer, taken to be a space ship far removed from the solar system, located among the nearby stars but at great distances from each of them, not moving with respect to the Sun, and equipped with standard cesium atomic clocks with which to measure time. This observer is clearly aware of Einsteinian time. The one hundred years which have elapsed on Earth since Einstein's birth would be recorded by the Herblock observer to be about 46 seconds longer due to the effects of gravitational potential difference (29 seconds from the Sun's potential and $\sim$2 seconds from the Earth's potential) and orbital motion of the Earth ($\sim$15 seconds). In the next section, I wish to recall and explain these fundamental effects on time which Einstein originally revealed to us by pure thought.

To continue with Einstein's life, Figure 3 shows his earliest known picture at the age of about 5. This is when he first saw a magnetic compass and began to wonder about its behavior. Figure 4 shows him in elementary school in Munich at the age of 10. He is second from the right in the front row. Figure 5 shows him at the age of 14. He had begun to read Euclidean Geometry two years earlier and a year later was to become a high school dropout because of dissatisfaction with the methods of teaching in his Munich gymnasium. He spent a happy year in Northern Italy where his family had moved, travelling and continuing his own studies. At the age of 16, he attempted to enroll in the Swiss Federal Institute of Technology (Eidgenössische Technische Hochschule) in Zurich, but failed the entrance examination overall, although he impressed the examiners with his knowledge and ability in physics and mathematics. These examiners recommended that he spend a year in the high school in Aarau, Switzerland, and then enter the ETH, since no entrance examination was required of graduates of Swiss secondary schools. Figure 6 shows the 16 year old Albert Einstein (far right, first row) in the Aarau classroom of an excellent teacher, Dr. Jost Winteler. It was during this period that Einstein began to think about what a beam of light would look like if somehow he were able to move fast enough to catch up with it. Figure 7 shows him as a student at the ETH in Zurich, where he succeeded in disappointing and alienating his professors by attending the required classes only sporadically while pursuing independent studies. (He particularly liked to spend time in the electricity and magnetism laboratory and in the study of Maxwell's Theory of the electromagnetic field, which was not then taught at the ETH). He passed the few required examinations only with the help of his friend, Marcel Grossmann, who was a model student, attending lectures and taking careful notes, which he lent to Einstein. The famous mathematician, Hermann Minkowski, who lectured at the ETH and later contributed the geometrical point of view to space-time and relativity, said,[1] on learning of Einstein's 1905 paper on restricted relativity, "Oh, that Einstein, always missing lectures - I really would not have believed him capable of it!" On another occasion, he referred to Einstein as "a lazy dog".

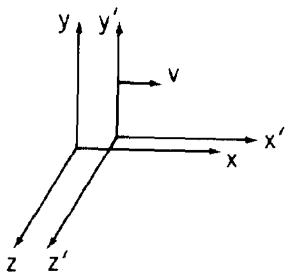
Because of these attitudes, which seem to have been shared by other members of the faculty, Einstein could not obtain an academic position or a permanent job of any sort for about a year and a half after he graduated. (As a teacher in a university, I reflect often, when looking at these pictures of Einstein during his student years, how easy it is to completely misjudge a student's true abilities. Conventional teaching in universities has changed little since Einstein's student days, and we have yet to react to his criticisms. A recent essay by Martin Klein[2] on Einstein and the Academic Establishment is particularly pertinent as are Einstein's own essays on education[3]). Finally, his good friend Marcel Grossmann prevailed on Grossmann's father who was acquainted with the director of the Swiss Patent Office in Bern to intercede and obtain an interview for Einstein. This led to employment there for seven fruitful years from 1902 until 1909. He had a natural aptitude for evaluating the feasibility of the patent applications, leaving time and energy to wonder about physics. Often in later life when asked for advice by young scientists, he would recommend, with his experience at the patent office in mind, that they not be dependent for their livelihood on the production of scientific results because of his concern with the corrupting influence of the need to be successful. In Figure 8, he is shown at the age of 26 at his desk at the Patent Office in the year 1905 when he published three remarkable papers in the Annalen der Physik: One introducing the light quantum into physics, one discussing the theory of the Brownian Motion and providing the clinching argument for skeptics of the existence of atoms, and the third, "On the Electrodynamics of Moving Bodies," containing his revolutionary insight about the relative nature of simultaneity and the non-absolute nature of time, the fundamental key to his restricted theory of relativity. About his years at the Patent Office, and Marcel Grossmann's help in obtaining the position, he wrote in a letter of condolence to Grossmann's widow in 1936, "...This saved my life; not that I would have died without it, but I would have been intellectually stunted", and, in the last year of his life, "The greatest thing that Marcel Grossmann did for me as a friend". There was, however, a third major thing that Grossmann did for Einstein. When he needed to learn differential geometry and the tensor calculus in order to give mathematical form to his idea of gravity as curved space-time, he turned to Grossmann who had specialized in the subject and was a member of the faculty of mathematics at the same ETH in Zurich when Einstein was a member of the physics faculty there from 1912 to 1914. Figure 9 shows Einstein in a playful mood at the California Institute of Technology in the early 1930's while Figure 10 shows him at the Institute for Advanced Study in Princeton where he spent the last twenty-two years of his life.

If these tidbits have stimulated you to want to learn more about Einstein's life, there is no better book to begin with than the brilliant biography by Professor Banesh Hoffmann, Albert Einstein, Creator and Rebel.[4] Many of the facts stated above, along with many of the photographs, were taken from this book. It has the added virtue of explaining Einstein's physics in a particularly clear way. Some of the remarks in the next sections of this talk follow the approach of Hoffmann.

Brief Review of Restricted ("Special") Relativity

This review will be much too concentrated for those people who may be encountering the ideas for the first time. However, I shall assume that most people in this audience have had some prior contact with the subject so that we can recall the fundamental concepts quickly.

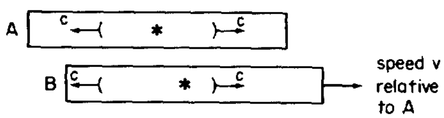
The restriction is to reference systems which move with respect to one another with constant velocity, and in any one of which a body at rest remains at rest and a freely moving body moves in a straight line - the so-called inertial systems. The adjacent diagram shows the conventional repre-



sentation of two such systems in relative motion. A fundamental postulate is that the laws of physics, electromagnetism and all other parts of physics, as well as dynamics, are to be the same in every inertial system. This has the consequence that no experiment carried out within an inertial system can distinguish it from any other such system. This is the restricted (special) Principle of Relativity.

The second fundamental postulate which Einstein identified had to do with the velocity of light in empty space. Light travels with a definite speed ($c \approx 3 \times 10^8$ meters per second) for every inertial observer which does not depend on the motion of its source.

These two postulates appear to be hopelessly in conflict. Consider the two inertial systems illustrated, each of which has a light source at

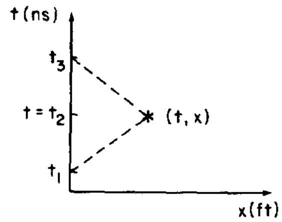


its center. When the two light sources are abreast of each other, let them flash simultaneously, sending out light wave fronts in the forward and back directions for observer A and for observer B. (These could be railroad cars, in Einstein's own example, or space ships, appropriate to our own epoch. For simplicity, we are limiting ourselves to only one dimension.) In his own system, A would see the pulses reaching the ends at the same time, but he would observe the backward travelling wave front reach the back end of system B before the forward travelling wave front reaches the front end of system B. How could observer B measure that the speed of light was c in both directions, as A would surely measure? Does this not violate the postulate of relativity, by which A and B should agree on the laws of physics?

You know well the resolution of this dilemma, which came to Einstein after many years of musing and bafflement. On awaking one morning during his time at the Patent Office, he sat bolt upright in bed with the realization that time is not absolute. The situation described is only "apparently irreconcilable". The simultaneity of separated events is relative to the inertial observer. Different inertial observers will not agree on whether two spatially separated events are simultaneous. For some observers, they may be simultaneous, but other observers will not even agree on which event occurs before the other. This fundamental realization about time has had profound consequences for all of physics!!

Einstein's Prescription for Comparison of Time (Clock Readings) for Separated Events

The following illustration shows Einstein's prescription for time comparison and affords the opportunity of introducing space-time diagrams:



Let the vertical axis measure time in nano-seconds and the horizontal axis measure distance (one dimension) in feet (~30 cm). Then the plot of a moving light pulse will make an angle of 45° since $c \sim 1$ foot per nano-second. The prescription is simple. Send out a pulse at time t_1, let it be reflected at the distant event, and return to the observer at time t_3. One assumes that the time of reflection assigned by the observer in midway between t_1 and t_3:

$$t = t_2 = t_1 + \frac{1}{2}(t_3 - t_1) = t_1 - \frac{1}{2}t_1 + \frac{1}{2}t_3$$
$$= \frac{1}{2}(t_1 + t_3) \qquad (1)$$

The same measurements of time will also yield the distance to the event by using the radar equation:

$$x = \frac{c}{2}(t_3 - t_1) \qquad (2)$$

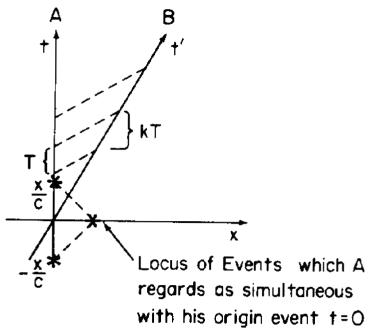
The diagram is called a Minkowski diagram after Einstein's distinguished professor of Mathematics at the ETH who developed this geometric way of looking at space-time in 1907.

Modern Observers and Minkowski Diagrams

The modern observer will be equipped with:

1. Atomic Clocks
2. Short Pulse Lasers
3. Fast Photo-detectors
4. Event Timers

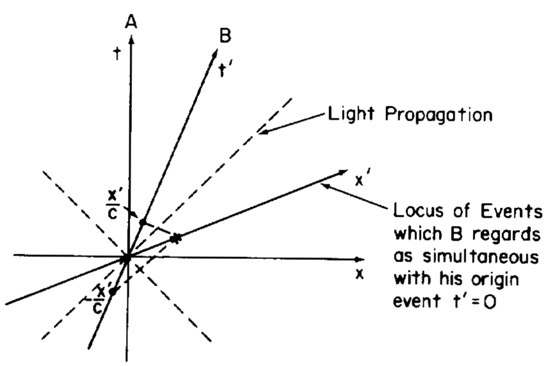
The k-Calculus. By sending and receiving short light pulses and recording the times (epochs) of such events, one can measure the space-time coordinates of distant events, as we have just seen. In addition, the technique lends itself to a very clear way of developing the conceptual structure of relativity, as was first done by Bondi.[5] Consider the following diagram in which the world line of B is represented in the space-time diagram of A.



Observer B has the same equipment as A, in particular a standard atomic clock. Because of the motion of B relative to A, light pulses emitted by A with a time interval T between them will be received by B with a stretched time interval kT because of the additional distance travelled by B between reception of pulses. This is just the[6] familiar Doppler effect. It is an easy exercise to show that the relativistic Doppler factor k is given by

$$k = \sqrt{\frac{1 + v/c}{1 - v/c}} \qquad (3)$$

The locus of the events which A regards as simultaneous with his origin event t = 0 as determined by the operation shows in the above diagram, a pulseless effect at t = -x/c being received back at t = x/c. This constitutes A's x-axis. If observer B carries through the same operation, as shown in the following diagram, emitting a pulse at t' = -x'/c and receiving it back at t' = x'/c,

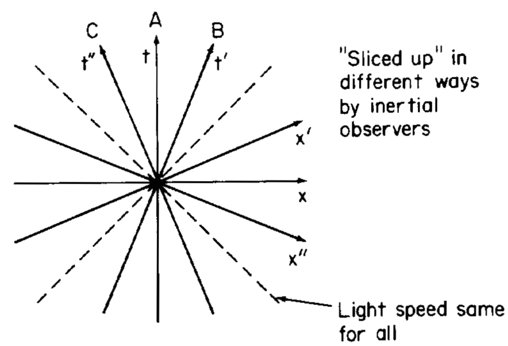


Space-Time

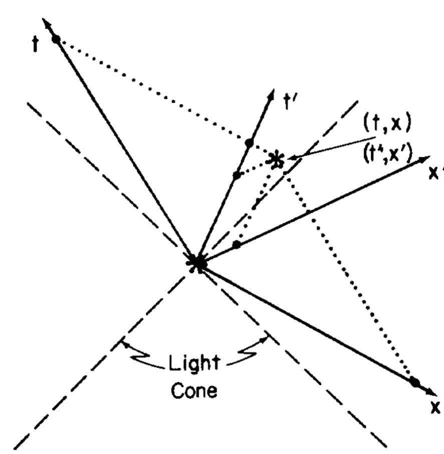
the locus of all such reflection are the events he regards as simultaneous with his origin even to t' = 0. (The same even t to which A assigns t=0). This x'-axis makes the same angle with respect to the locus of light propagation, the 45° line, as the t'-axis. The light propagation lines, shown dashed in the figure, are the same for both observers because of the invariance of the speed of light. They are often called the light cone.

The geometrization of space-time, as pointed out by Minkowski in 1907, amounts to this: There is an absolute space-time, which is "sliced up" by different inertial observers in different ways, as shown in the following diagram (for one space dimension)

Minkowski's Absolute Space-Time (1907)



Some reference event is chosen and a light cone is associated with it. If this reference event is chosen as the origin event for several inertial observers, their time and space axis are plotted as shown. Observer C is moving to the left with respect to observer A. The distance in the diagram along the time and space axes which corresponds to unity will be different for each inertial observer[7]. The coordinates for an event are determined by parallel projection onto the time and space axes for any inertial observer, as shown in the following diagram



Minkowski showed a very remarkable property: even though $t' \neq t$ and $x' \neq x$ in the above diagrams, there is an expression which is the same for both observers, namely[8]

$$s^2 \equiv c^2 t^2 - x^2 = c^2 t'^2 - x'^2 \tag{4}$$

This is called the square of the <u>invariant interval</u> between the origin event (0,0) and the event (t,x) or (t', x'). For any two events whose separation in time is Δt or $\Delta t'$ and whose separation in space is Δx^2 or $\Delta x'$, it is easily shown that

$$(\Delta s)^2 = c^2(\Delta t)^2 - (\Delta x)^2 = c^2(\Delta t')^2 - (\Delta x')^2 \tag{5}$$

Even though $\Delta t \neq \Delta t'$ and $\Delta x \neq \Delta x'$, all inertial observers will get the same result for $(\Delta s)^2$ as defined by equation (5)! In three spatial dimensions, $(\Delta s)^2$ becomes

$$\begin{aligned}(\Delta s)^2 &= c^2(\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2 \\ &= c^2(\Delta t')^2 - (\Delta x')^2 - (\Delta y')^2 - (\Delta z')^2\end{aligned} \tag{6}$$

and continues to be invariant for all inertial observers.

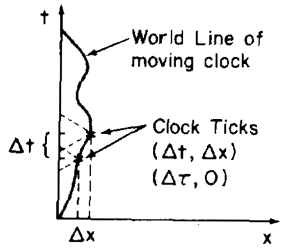
This is an extremely important result because it provided the mathematical basis for Einstein's later development of his theory of gravity - a curvature of the flat space-time that we have been discussing in terms of the preceding diagrams - as we shall discuss later on. Minkowski characterized his development of the geometry of space-time during an address to the 80th Assembly of German Natural Scientists and Physicians in Cologne on the 21st of September, 1908 in the following famous excerpt:

> "The views of Space and Time which I wish to lay before you have sprung from the soil of experimental physics, and therein lies their strength. Henceforth, space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality."

He was, of course, talking about the slicing up of space-time by different inertial observers, accompanied by the invariance of the interval, as we have just discussed.

<u>The Effect of Motion on Clocks</u>

This is most readily described by using the invariant interval. Consider the following Minkowski space-time diagram which represents the motion of a clock with respect to some arbitrary inertial observer whose coordinate time and space axes are displayed. The world line of the moving clock is just the locus of the events at which



Reading of moving clock is its own time. Proper Time. Denote by τ.

it is present. The curvature of the world line shows that it experiences accelerations. Two neighboring events along the world line of the clock are shown representing "ticks" of the clock (every second, or better, every nanosecond). With respect to the inertial observer, they have a time separation Δt and a spatial separation Δx. For the moving observer accompanying the clock, the time separation between ticks is the time interval actually recorded by the moving clock, which is called its <u>proper time</u> interval and given the symbol $\Delta\tau$. The spatial separation between the ticks for the observer moving with the clock will be zero, since the clock is always at the origin of the moving coordinate system. If the two tick events are close together, the moving observer can be regarded as an inertial observer since his instantaneous velocity v will change very little during the interval between ticks. Then, denoting the time and space measurements of the moving observer with primes,

$$(\Delta s)^2 = c^2(\Delta t)^2 - (\Delta x)^2 = c^2(\Delta t')^2 - (\Delta x')^2 \tag{7}$$

But

$$\begin{aligned}\Delta t' &= \Delta\tau \\ \Delta x' &= 0\end{aligned} \tag{8}$$

and

$$\Delta x = v\Delta t \tag{9}$$

Equation (7) thus becomes

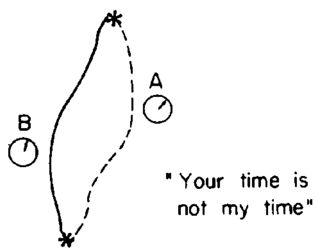
$$\begin{aligned}c^2(\Delta\tau)^2 &= c^2(\Delta t)^2 - (v\Delta t)^2 \\ &= (c^2 - v^2)(\Delta t)^2\end{aligned} \tag{10}$$

or

$$\begin{aligned}(\Delta\tau)^2 &= \left(1 - \frac{v^2}{c^2}\right)(\Delta t)^2 \\ \Delta\tau &= \sqrt{1 - \frac{v^2}{c^2}}\ \Delta t\end{aligned} \tag{11}$$

relating the <u>proper time increment</u> $\Delta\tau$ of the moving clock to the <u>coordinate time increment</u> Δt of the inertial observer. Note that this important result has been obtained with very little mathematics. The author has successfully taught these ideas to introductory physics students, given several weeks for them to be absorbed gradually.

If two identical clocks are synchronized to read the same when they are together, and observed to have the same rate when together, they will not exhibit the same reading after being separated and experiencing different routes in space-time before being brought together again, even though their rates will again be the same. The situation is illustrated below for clocks A and B.



The difference in proper times for clocks A and B will be

$$\tau_A(\text{final}) - \tau_B(\text{initial}) = \int d\tau_a = \int \sqrt{1 - v_A^2/c^2}\;dt$$

$$\tau_B(\text{final}) - \tau_B(\text{initial}) = \int d\tau_b = \int \sqrt{1 - v_B^2/c^2}\;dt \qquad (12)$$

Where t represents the coordinate time of some inertial observer and v_A and v_B are the instantaneous velocities of A and B with respect to that inertial observer. The elapsed proper time will be different for clocks A and B since their histories or routes are different. There is a route dependence of elapsed proper time. Colloquially, to paraphrase a once popular song, "Your time is not my time"!

Note in equation (12) that there is no explicit dependence on the acceleration or higher derivatives of the motion of the clocks, but only the instantaneous velocity. This is sometimes referred to in the literature of relativity as the "clock hypothesis". There clearly must be some acceleration for the clocks to separate and be brought back together again. Any real clock will be influenced by acceleration to some extent. For example, a watch dropped on a hard floor from a sufficient height will probably stop running! However, it is possible in real situations to keep accelerations small enough to avoid significant rate changes by careful packaging to reduce shocks and vibrations and by sufficiently slow motions of the vehicle (e.g. aircraft) carrying the clock. The important point is that there is no specific dependence on the instantaneous acceleration in the theory predicting the elapsed proper time for an arbitrary motion of a clock in space-time.

<u>Einstein's 1905 Prediction</u>

In the 1905 paper, "On the Electrodynamics of Moving Bodies", referred to earlier, Einstein made the following statement after developing his ideas about time:

> "Thence we conclude that a balance-clock* at the equator must go more slowly, by a very small amount, than a precisely similar clock situated at one of the poles under otherwise identical conditions.
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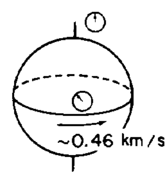
>
> *Not a pendulum clock, which is physically a system to which the earth belongs. This case had to be excluded."

The situation is sketched below. The equatorial



surface velocity is about 0.46 meters/second. Equation (12) evaluated in an inertial frame (non-rotating) with origin at the center of the earth yields, to first order in v^2/c^2.

$$\tau(\text{final}) - \tau(\text{initial}) \simeq \int \left(1 - \frac{v^2}{2c^2}\right) dt \simeq t - \frac{v^2}{2c^2}\,t \qquad (13)$$

At a pole, $v = 0$, but at the equator $v^2/2c^2 = 1.18 \times 10^{-12}$. If t is one day, the equatorial clock would run slow with respect to the polar clock by about 102 nanoseconds.

If atomic clocks had existed in 1905 with the stability we have today ($\sim$2 ns/day) so that the prediction could have been tested, it would have been found to be wrong! Why? Because the effect of gravity had not been included! Two years were to go by before Einstein discovered the Principle of Equivalence in 1907 and drew the conclusion about the influence of gravitational potential on time. The oblateness of the spinning earth causes a decrease in the gravitational potential as one moves from the equator to a pole, getting nearer to the center of the earth. It is remarkable that the effect of this change on clocks is predicted to offset exactly the effect of the change of surface velocity, so that the correct prediction is a null effect. We have performed an experiment recently, transporting clocks from Washington, D.C. to Thule, Greenland and back, which supports this null prediction, and will describe it later.

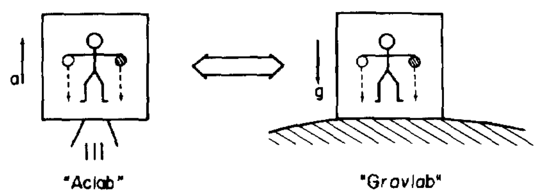
The Principle of Equivalence
(Einstein's "Happiest Thought")

Let us now turn to the incorporation of gravity into the structure of space-time which in Einstein's hands produced the General Theory of Relativity - no longer restricted to inertial frames of reference. It is my experience that the best way to understand Einstein's theory of gravity is through the historical route actually followed by Einstein. The physical ideas came before the full mathematical formulation of curved space-time involving the tensor calculus, which took eight more years to develop. The key idea came to Einstein in 1907 when he was working on a summary essay concerning the special theory of relativity for the yearbook for Radioactivity and Electronics. He described his train of thought in an essay written in 1919, "The Fundamental Idea of General Relativity in its Original Form", which is not yet published, but an excerpt was printed in the New York Times[9] in 1972 when the planned editing and publication of all his papers was announced.

> "I tried to modify Newton's theory of gravitation in such a way that it would fit into the theory. Attempts in this direction showed the possibility of carrying out this enterprise, but they did not satisfy me because they had to be supported by hypotheses without physical basis. At that point, there came to me the <u>happiest thought of my life</u> [emphasis added] in the following form:
>
> Just as in the case where an electric field is produced by electromagnetic induction, the gravitational field similarly has only a relative existence. Thus, for an observer in free fall from the roof of a house there exists, during his fall, no gravitational field - at least not in his immediate vicinity. If the observer releases any objects, they will remain, relative to him, in a state of rest, or in a state of uniform motion, independent of their particular chemical and physical nature.* The observer is therefore justified in considering his state as one of "rest".
>
> The extraordinarily curious, empirical law that all bodies in the same gravitational field fall with the same acceleration immediately took on, through this consideration, a deep physical meaning. For if there is even one thing which falls differently in a gravitational field than do the others, the observer would discern by means of it that he is in a gravitational field, and that he is falling in it. But if such a thing does not exist - as experience has confirmed with great precision - the observer lacks any objective ground to consider himself as falling in a gravitational field. Rather, he has the right to consider his state as that of rest, and his surroundings (with respect to gravitation) as field-free.
>
> The fact, known from experience, that acceleration in free fall is independent of the material, is therefore a mighty argument that the postulate of relativity is to be extended to coordinate systems that are moving non-uniformly relative to one another.
>
> ---
>
> *In this consideration one must naturally neglect air resistance."

The following simple diagrams illustrate the point made by Einstein and allow profound consequences to be drawn with little or no mathematics! Consider a laboratory falling freely as shown below. Objects released with no initial velocity

will remain at rest. If the initial velocity is not zero, the path of the object will be a straight line. This is now very familiar to us from television and movies of the U.S. astronauts in Skylab and the Apollo spacecraft, and the Soviet cosmonauts in Salyut and the Soyuz spacecraft. <u>The freely falling spacecraft constitutes a true (local) inertial system.</u> If one imagines the spacecraft in a region of space free of gravity but subject to the acceleration produced by a rocket engine, as shown on the left below (the "Aclab" of Banesh Hoffmann), the observer inside will experience effects similar to those in a stationary lab-



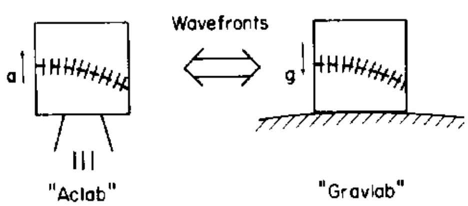
oratory on the surface of a body like the earth where there is an acceleration of gravity g. In both cases, objects released will move to the floor with accelerated motion. In the "Aclab" case, the floor accelerates to the released object, so that any object, regardless of its composition, will behave in the same way. In the "Gravlab" case, it has been measured with increasing accuracy by Galileo, Newton, Eötvös, Dicke, and Braginsky, that all bodies at the surface of the earth fall with the same acceleration. The latest measurements,[10] using torsion balance techniques to compare aluminum and gold at Princeton University and to compare aluminum and platinum at Moscow State University give

$$\left|\frac{g_{al} - g_{au}}{g}\right| < 10^{-11} \quad \text{(Dicke, 1964)}$$

$$\left|\frac{g_{al} - g_{pt}}{g}\right| < 10^{-12} \quad \text{(Braginsky, 1972)} \qquad (14)$$

Implications for Light Propagation

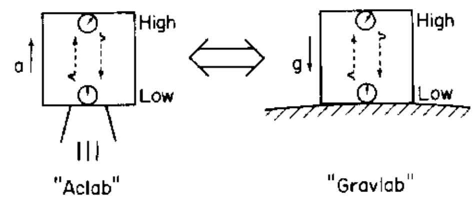
Einstein proposed that the equivalence idea should hold, not only for dynamics, but for all of physics, and, in particular, for electromagnetic phenomena, including light. If this is the case, one can draw some conclusions without using any mathematics at all as illustrated in the following diagram. Since the "Aclab" is accelerating with



respect to an inertial system, and a beam of light would propagate in a straight line in that system, to an observer in the "Aclab", the light would appear to follow a curved path. If the equivalence to the "Gravlab" is correct, the prediction would be that light follows a curved path when propagating in a gravitational field. In addition, by noting that the wave fronts associated with the light beam must move like ranks of soldiers when turning, the outer soldiers moving faster than the inner ones, the conclusion follows that the speed of light should increase with height in a gravitational field!!

Implications for the Rate of Clocks

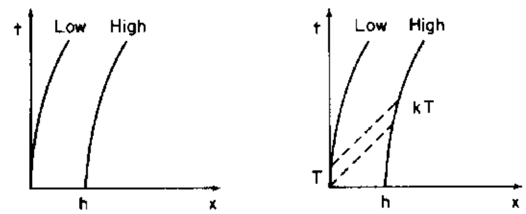
Imagine two atomic clocks of identical construction to be at the top and bottom of an "Aclab" as shown below, their readings being compared by modern observers equipped with short pulse lasers, fast photo-detectors, and event timers as discussed earlier. The rates can be calculated using the



techniques of restricted relativity discussed earlier (accelerated motion can be analyzed by using a succession of inertial frames having the instantaneous velocity of the accelerated object). The conclusion, as we will show below, is that the high clock will run fast with respect to the low clock. Therefore, by the principle of equivalence, in the "Gravlab", the high clock will run fast with respect to the low clock. A clock's rate is predicted to depend on its position in a gravitational field!!

Since we have presented earlier all the mathematical machinery needed to draw this conclusion, let us run briefly through the analysis. Construct a space-time diagram for the low and high observers in the "Aclab" referred to time and space axes of an inertial observer, as shown on the left below. The curvature of the world lines represents

Comparison of Clocks in "Aclab"



their acceleration with respect to the observer establishing the time and space coordinates. They are separated by a vertical distance h when t = 0. Let two successive light pulses be emitted by the low observer separated by a time interval T on his clock. The pulses will be received by the high observer separated by a time interval kT in accordance with our earlier discussion of the Doppler factor, equation (3). The velocity which should be used to evaluate k in that of the high observer when the first pulse reaches him.

$$v = at \simeq ah/c \qquad (15)$$

where a is the acceleration of the "Aclab" and the approximation is made that the separation is still h at this later time so the transit time ≃ h/c. It is further assumed that during the time T the velocity will not change appreciably.

Then

$$k = \sqrt{\frac{1 + v/c}{1 - v/c}} \approx \sqrt{1 + 2v/c} \approx \sqrt{1 + \frac{2ah}{c^2}} \qquad (16)$$

Now, by the Principle of Equivalence, a = g, the acceleration of gravity, so k becomes

$$k = \sqrt{1 + \frac{2gh}{c^2}} \qquad (17)$$

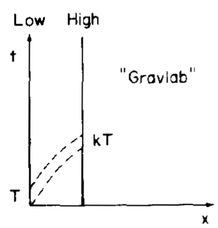
Recall that gh is just the Newtonian potential difference ϕ if h is small,

$$\phi \equiv gh \qquad (18)$$

so that

$$k = \sqrt{1 + \frac{2\phi}{c^2}} \qquad (19)$$

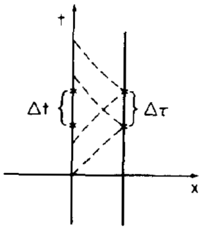
By the Principle of Equivalence, the situation would be as shown in the following space-time diagram. The low and high clocks are not moving



so their world lines are straight. The light propagation lines are curved because of the increase of the speed of light with height. The time interval T becomes stretched to kT, where

$$k = \sqrt{1 + \frac{2gx}{c^2}} \qquad (20)$$

This circumstance is what can be called the "curvature of time." The elapsed proper time of a clock depends on where it is located in the gravitational field and differs from the elapsed coordinate time established by an observer, at the surface of the earth, for example, with the aid of light signals. The following diagram shows the relation for vertical distances.



The observer on the ground at x = 0 can establish the coordinates (t, x) of any event by sending out a light pulse to be reflected back at the time of the event and received at a later time. The time he assigns to the event is halfway between his emission and reception events as described by equation (1). (This is still valid even though the speed of light is not constant in a gravitational field; the time required for the light pulse to go out is the same as that for it to return). If the procedure is repeated an interval of coordinate time Δt later, it will define an interval of proper time $\Delta\tau$ at the higher position, as shown in the diagram. The essential property is that

$$\Delta t \neq \Delta\tau \qquad (21)$$

Time Curvature

It is possible to generalize the expression for the square of the invariant interval in restricted relativity, equations (5) and (6), to allow for this effect, and that is just what Einstein did, writing now

$$\Delta s^2 = \left(1 + \frac{2gx}{c^2}\right) c^2 (\Delta t)^2 - (\Delta x)^2 \qquad (22)$$

For three spatial dimensions, and using the Newtonian potential ϕ to include non-uniform gravitational fields

$$\Delta s^2 = \left(1 + \frac{2\phi}{c^2}\right) c^2 (\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2 \qquad (23)$$

Einstein retained the interpretation of Δs as measuring the interval between two events in space-time, so for the interval between two ticks of a clock,

$$\Delta s = c\Delta\tau \qquad (24)$$

where $\Delta\tau$ is the interval of proper time recorded by the clock. If the clock is not moving

$$(\Delta s)^2 = c^2 (\Delta\tau)^2 = \left(1 + \frac{2\phi}{c^2}\right) c^2 (\Delta t)^2 \qquad (25)$$

or

$$\Delta\tau = \sqrt{1 + \frac{2\phi}{c^2}}\ \Delta t \qquad (26)$$

which is the same as equation (19), giving the relation between increments of proper time and coordinate time. If the clock is moving,

$$\Delta x = \upsilon_x \Delta t, \ \Delta_y = \upsilon_y \Delta t, \ \Delta z = \upsilon_z \Delta t \qquad (27)$$

and equation (23) becomes

$$(\Delta s)^2 = c^2(\Delta\tau)^2 = \left(1 + \frac{2\phi}{c^2}\right) c^2 \, (\Delta t)^2$$
$$- \{\upsilon_x^{\,2} + \upsilon_y^{\,2} + \upsilon_z^{\,2}\} \, (\Delta t)^2 \tag{28}$$

which leads to

$$\Delta\tau = \sqrt{1 + \frac{2\phi}{c^2} - \frac{v^2}{c^2}} \;\; \Delta t \tag{29}$$

as the relation between proper time and coordinate time increments for a clock in motion.

As Einstein developed these concepts during the years 1907 - 1915, and especially during the collaboration with Marcel Grossmann from 1912 - 1914, he realized there would also be a curvature of space produced by a gravitational field. This would be represented by coefficients of the $(\Delta x)^2$, $(\Delta y)^2$, and $(\Delta z)^2$ terms of the expression for $(\Delta s)^2$ which would also be functions of position, just as the coefficient of the $c^2(\Delta t)^2$ term. The values of these coefficients are determined by his field equations for a given distribution of matter. For a spherically symmetric mass distribution, the famous Schwarzschild solution of the field equations is

$$(\Delta s)^2 = \left(1 + \frac{2\phi}{c^2}\right) c^2(\ t)^2 - \frac{(\ r)^2}{\left(1 + \frac{2\phi}{c^2}\right)}$$
$$- r^2 \cos^2\beta(\Delta\alpha)^2 - r^2 \, (\Delta\beta)^2 \tag{30}$$

where

$$\phi = - \frac{GM}{r} \tag{31}$$

with M the mass of the central body, r the radial distance, α the longitude, and β the latitude. For motions in weak gravitational fields such that

$$\frac{|\phi|}{c^2} < < 1, \tag{32}$$

and for slow motions,

$$\frac{v^2}{c^2} < < 1, \tag{33}$$

one can neglect the coefficients of the spatial increments, and one is left with the equation (23) containing only the curvature of time.

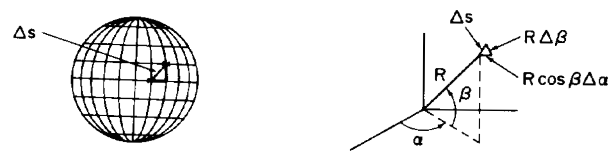
Under the conditions of low velocity and weak gravity expressed by equations (32) and (33), one can approximate equation (29) as

$$\Delta\tau \simeq \left(1 + \frac{\phi}{c^2} - \frac{v^2}{2c^2}\right) \Delta t \tag{34}$$

All of the experiments with atomic clocks which have been done recently, and which we will describe below, are a testing of the relationship (34) between proper time and coordinate time.

<u>Analogy of Time Curvature to the Curvature of a Sphere</u>

Consider a sphere, like the earth, whose curved surface possesses a coordinate system of latitude and longitude, as shown in the following diagrams. The coordinates of longitude α runs through the range 0 to 360.



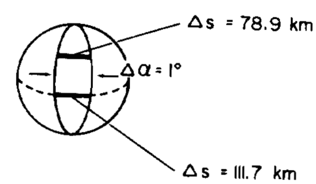
The coordinate of latitude runs from -90° to 90°. R is the radius of the sphere. What is the distance Δs between two neighboring points on the surface of the sphere, differing by $\Delta\alpha$ and $\Delta\beta$? It is <u>not</u> given by

$$(\Delta s)^2 = (\Delta\alpha)^2 + (\Delta\beta)^2 \quad \text{(Wrong!)} \tag{35}$$

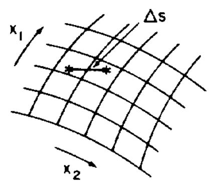
but rather by

$$(\Delta s)^2 = R^2 \cos^2 \beta(\Delta\alpha)^2 + R^2 \, (\Delta\beta)^2 \tag{36}$$

There are <u>metric coefficients</u>, which depend on position, multiplying the square of the coordinate differentials to give the true measure of length, or <u>proper length</u>, between the points. The actual proper length will be different for different locations on the sphere, even though the coordinate differentials are the same. For example, consider $\Delta\alpha = 1^\circ$ and $\Delta\beta = 0$, as shown in the following exaggerated diagram.



At the equator, Δs will be 111.7 km, while at the latitude of 45°, Δs will be 78.9 km. The relation between the proper time increment $\Delta\tau$ and coordinate time increment Δt in curved space-time is very closely analogous to the relation between the proper length increment Δs and the coordinate of longitude increment $\Delta\alpha$ on the curved surface of the sphere. The coefficients in relativity are also called metric coefficients. For a network of curvilinear coordinates, x_1 and x_2, on an arbitrary curved surface as shown the proper length Δs is



expressed as:

$$(\Delta s)^2 = g_{11}(\Delta x_1)^2 + 2g_{12}(\Delta x_1)(\Delta x_2) + g_{22}(\Delta x_2)^2 \quad (37)$$

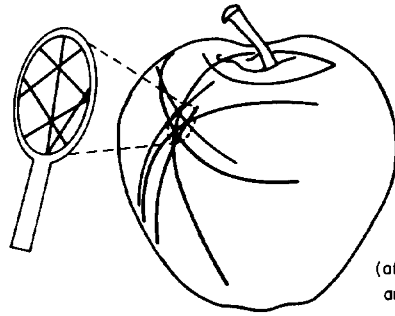
The great mathematician Gauss made many contributions to the differential geometry of two-dimensional curved surfaces, in particular showing that the curvature can be calculated from the variation of the metric coefficients g_{ij} on the surface only.[11] These results were generalized to an arbitrary number of coordinates describing n-dimensional curved space by Riemann. This Riemannian geometry furnished many mathematical tools which Einstein and Grossmann used in the final form of General Relativity.

Brief Summary of the Curved Space-time Theory of Gravity

In the curved space-time produced by the presence of matter, the metric coefficients in the expression for $(\Delta s)^2$ being determined by the Einstein field equations, a free object will move in such a way that if one imagines it carrying a clock, the path it follows in space-time will make its elapsed proper time a maximum. In technical terms, it will follow a so-called geodesic path which is the analog of a great circle on a sphere. Locally, the path will be as straight as possible in the curved space-time. In the words of Professor John Wheeler,

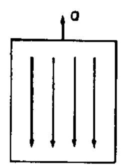
> "Matter tells space-time how to curve;
> Curved space-time tells objects how to move."

This is graphically represented by the following picture and analogy taken from the book Gravitation by Misner, Thorne, and Wheeler.[12] The curved surface of the apple may be thought of as caused by the stem (analog of mass). The ants moving on the curved surface try to move locally as straight as possible (geodesics). The result is that the ants end up moving in curved paths about the stem (planet orbiting the sun).



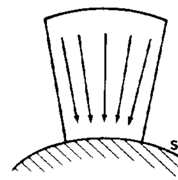
(after Misner, Thorne, and Wheeler)

There is another way of looking at free motion in the curved space-time produced by masses. Recall from Einstein's essay that the Principle of Equivalence allows the local cancellation of the effects of gravity by going to a freely falling frame of reference. It can only be local because gravitational fields in the real world are not uniform, as the following diagrams indicate. The extent of the local freely falling frame in both time and space will depend on the accuracy with



AcLab

Paths of dropped objects are parallel

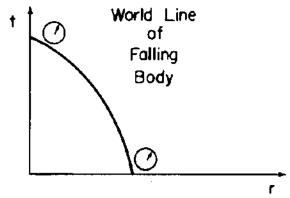


Large GravLab

Converging Paths of dropped objects

which measurements are made. These local freely falling frames obviously are local inertial systems in that freely moving objects will move in straight lines as discussed earlier. Inertial systems must be local because only non-uniform gravitational fields exist in the real world. The inertial systems of large extent as discussed in restricted relativity are a fiction. An object moving freely in a non-uniform gravitational field (curved space-time) can be thought of as moving in straight lines in the flat space-time of restricted relativity in each of a succession of local freely falling systems which differ in both direction and velocity of motion. It is clear from equations (12) and associated diagrams that such straight line motion in an inertial system between two events in space-time produces a proper time difference greater than curvilinear motion, since there is some inertial system in which it would be at rest. Hence, the conclusion that free motion in curved space-time follows a path which produces a maximum proper time difference. Bertrand Russell[13] has referred to this as the "Principle of Cosmic Laziness."

Einstein's theory of gravity as curved space-time does away with the concept of Newtonian gravitational forces. A body falls to the earth because it follows the locally straightest path in curved space-time. We have seen that for low velocities and weak fields, as exist for most motions on earth, the primary curvature is that of time. Thus, from the Einsteinian point of view, bodies fall because of the properties of time and the behavior of clocks which we are discussing. The essential property is the relation between proper time and coordinate time, equation (34). The curved world line of a falling body results from the elapsed proper time associated with the body

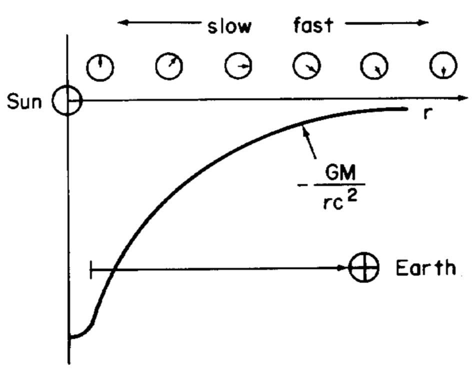


Curved Space-Time

$$\tau(\text{final}) - \tau(\text{initial}) = \int d\tau \approx \int \left(1 + \frac{\phi}{c^2} - \frac{v^2}{2c^2} \right) dt \qquad (38)$$

having a maximum value for that path as compared with other paths between the same initial and final events.

The following diagram shows a plot of gravitational potential ϕ/c^2 for the sun as a function of distance from its center, along with a schematic representation of the change in rate of clock as a function of position. This is one way of indicat-



ing the curvature of time. At the surface of the sun $\phi/c^2 \simeq -2 \times 10^{-6}$. Using the value at the earth of the solar gravitational potential, the earth gravitational potential ($\phi/c^2 \simeq -7 \times 10^{-10}$ at its surface) and the velocity of the earth in its orbit around the sun ($\sim$30 km/s), the additional 46 seconds since Einstein's birth as observed by Herblock's distant cosmic observer, (whose proper time will be the same as coordinate time), was calculated using equation (38) for the freely falling earth.

In the popular literature on general relativity, the curvature of space is referred to almost exclusively since it is somewhat easier to visualize than the curvature of time. However, this is quite misleading, since the curvature of time leads to all of Newtonian physics for low speeds and weak fields in Einstein's theory, and in this sense can perhaps be regarded as the primary curvature. One should regard most of these references to the curvature of space as shorthand for the curvature of space-time. Now that it is possible to measure accurately with modern atomic clocks, as described in the next sections, the remarkable properties of Einsteinian time, and as the properties are, of necessity, used more and more in global timekeeping systems, we can hope for more intuitive understanding of the physics of general relativity.

Experimental Measurements

Introduction

The experiments which will mainly be described are those of the author and his collaborators with modern cesium beam atomic clocks in aircraft and the experiment with a hydrogen maser in a rocket probe conducted by Vessot and Levine. These experiments were designed to measure primarily the effects of gravitational potential, although the effects of motion were necessarily present and were also measured. They used macroscopic oscillators controlled by atomic resonances. Before describing them, it is appropriate to mention for completeness some of the earlier observations using direct radiation from atoms and nuclei.

Optical Spectroscopy

Solar Redshift Observations. The first tests[14] of the properties of time as affected by gravitational potential were sought in the shift toward the red of the frequency of optical radiation emitted by atoms on the sun and received and compared on earth with radiation from similar atoms. The potential difference would predict a shift $\Delta f/f \sim 2 \times 10^{-6}$. It was very difficult to establish this value with any confidence because of lack of accurate knowledge of pressure-induced frequency shifts in the sun's atmosphere and the complications of Doppler shifts due to turbulent motions of the emitting gas atoms and the surface velocity of the sun due to its rotation. A brief summary with references of these older observations and the attempts to interpret them is given by Pauli.[15]

The choice of atoms whose spectral lines are little affected by pressure, and the use of rapid switching in the earth laboratory from solar radiation to laboratory sources has allowed measurements with a believable accuracy of about 5%. The experiments were done at Princeton University by J. Brault (1963)[16], in France by J. Blamont and F. Roddier (1965)[17], and in the U.S.A. at Oberlin College by J. Snider (1972)[18].

White Dwarf Redshifts. These observations have suffered from lack of precise knowledge of the size and mass of the objects from which to calculate the surface gravitational potential. Sirius B provides the comparison most widely known, although there are others, such as 40 Eridani B. The accuracy is probably no better than 15 to 20%.[19,20]

Moving Atoms. The first controlled measurements of the relativistic effects of motion were carried out by H.E. Ives and G.R. Stilwell (1941).[21] They measured the "transverse Doppler effect" (that is, the difference between coordinate time and proper time by examining the light from excited atoms moving in a collimated beam, much care being taken to average out the first order Doppler effect. The accuracy of comparison with theory is difficult to assess -- perhaps 5 to 10%. Similar experiments have been performed by G. Otting (1939).[22] H.L. Mandelberg and L. Witten (1962)[23] quote an accuracy of 5% in their more recent version of the experiment.

Using a beam of high velocity Na atoms and laser techniques for the cancellation of first order Doppler effects, the "time dilation" has been recently measured by J.J. Snyder and J.L. Hall (1975)[24] to 0.5%.

With Helium-Neon lasers stabilized by Methane, the expected shift of frequency of 0.542 per degree in the Methane due to the change of rms velocity with temperature has been observed with an uncertainty of 10% by S.N. Bagayev and V.P. Chebotayev (1972).[25]

Mössbauer Gamma Ray Spectroscopy

Effect of Gravitational Potential. The discovery by Mössbauer that the frequency shift associated with the recoil of the nucleus when emitting or absorbing a γ-ray photon could be very small when the nucleus transferred its momentum to the solid of which it was a part opened the way to very high resolution γ-ray spectroscopy. The application of the technique to a terrestrial measurement was first made by R.V. Pound and G.A. Rebka (1960)[26] using a 22.6 meter vertical path in a tower at Harvard University. The potential difference produces a frequency shift $\Delta f/f \sim 2.5 \times 10^{-15}$, which was measured with about 10% accuracy. Later Pound and J. Snider (1965)[27] improved the measurements to an accuracy of about 1%.

Motional Effects. The temperatures of the materials hosting the γ-ray emitting and absorbing nuclei in the above experiments need to be measured with precision and kept closely equal to prevent the motional effects on time of the lattice vibrations from masking the gravitational effect. Complications of solid state physics and temperature measurement do not allow a very accurate measurement, but the temperature dependence of the shift in resonant frequency provides evidence for the difference between coordinate time and proper time. The Mössbauer technique has been used to demonstrate the effect of velocity on time in centrifuge experiments by H.J. Hay, J.P. Schiffer, T.E. Cranshaw, and P.A. Egelstaff (1960);[28] by W. Kündig (1963);[29] and also by K.C. Turner and H.A. Hill (1964)[30] while studying other aspects of relativity. The centrifuge technique has produced an accuracy of comparison with theory of about 1%.

The very large accelerations associated with these motional effects, as well as the acceleration of the muons in storage rings discussed below, have no intrinsic effect on the relation between proper time and coordinate time as given in equation (11).

Astronomical Observations from the Moving Earth

The analysis of many types of observations involves the transformation from a reference system "attached" to the center of mass of the solar system in which calculations are more easily done, to the reference system associated with the earth. Such observations include the measurement of changing pulsar periods, very long baseline interferometry, lunar laser ranging, deep space tracking of spacecraft, and planetary ranging. All involve the use of stable atomic clocks on the earth. The difference between proper time on the earth and coordinate time in the solar system is therefore essential in the analysis of the results of the observations. A particularly clear discussion of the transformation is given by J.B. Thomas (1976).[31]

High Energy Physics

Charged Pion Lifetime. This has been measured by A.J. Greenberg, et al, (1969)[32] in a beam from the Lawrence Radiation Laboratory's 184-inch cyclotron where the factor $(1 - v^2/c^2)^{-\frac{1}{2}}$ had the value 2.4. The accuracy of the lifetime determination compared with the previously measured value of 26 nanoseconds for the lifetime at rest agrees with the relativistic prediction to 0.4%.

Moving Muons. The difference between coordinate time intervals and proper time intervals has been measured using the decay of muons. The lifetime of rapidly moving muons increased by the factor $(1 - v^2/c^2)^{-\frac{1}{2}}$. The most accurate measurement, about 2%, has been made in a storage ring at Center for Nuclear Research (CERN) in Geneva by Farley, et al, (1966).[33] The factor $(1 - v^2/c^2)^{-\frac{1}{2}}$ had a value of 12. Earlier, but much less accurate, measurements had been made as early as 1941 by Rossi and Hall[34] by studying as a function of altitude the survival of muons produced by cosmic rays in the upper part of the earth's atmosphere. They would not survive to sea level except for the relativistic effect since their life time at rest is only about 2 microseconds.

Relativistic Dynamics. The design of particle accelerators and the analysis of high energy experiments uses concepts of relativistic energy and momentum which are based squarely on the invariance of the interval, equation (6) above. Thus, high energy physics makes continuing use of the Einsteinian concept of time.

Atomic Clocks Carried in Commercial Aircraft on Around the World Flights

In October of 1971, J.C. Hafele and R.E. Keating[35] first demonstrated the relativistic effects on time for macroscopic atomic clocks by carrying an ensemble of four commercial cesium beam clocks (Hewlett-Packard Model No. 5061) belonging to the U.S. Naval Observatory around the world on scheduled airline flights, first in the eastward direction, and six days later in the westward direction. The combination of the surface velocity of the earth due to its rotation with the velocity of the jet aircraft leads to the prediction of an asymmetry in the relativistic effects for the different circumnavigation senses. The gravitational potential effect due to the altitude of the aircraft needs to be included. The following table gives their published values of the predicted effects:

Effect	Eastward	Westward
Potential	144 ± 14 ns	179 ± 18 ns
Velocity	-184 ± 18 ns	96 ± 10 ns
Net	- 40 ± 23 ns	275 ± 21 ns
Trip duration	65.4 hrs.	80.3 hrs.

The uncertainties are from a lack of sufficiently detailed knowledge of the velocity, altitude, and position of the airplanes during the flights.

The ensemble of clocks were intercompared to 1 ns once an hour in order to identify any rate changes of individual clocks with respect to the average. Several such rate changes were identified and corrected for in arriving at the measured results. The systematic error in this procedure is given as ±30 ns. Corrections for temperature and pressure changes were not made. The measured values of the average time difference with respect to the stay-at-home clocks at the U.S. Naval Observatory are given as:

Eastward -59 ns
Westward ±273 ns

The comparison with the predictions seems to show an uncertainty of about 13% for the westward direction, but much worse for the eastward direction. It is difficult to assign an uncertainty for the comparison of the individual potential and velocity effects, but their existence is certainly demonstrated.

Atomic Clocks Carried in an Aircraft on Local Flights with Radar Tracking and Laser Pulse Time Comparison

Participants. These experiments were conducted[36] by the author and L.S. Cutler, R.A. Reisse, R.E. Williams, J.D. Rayner, C.A. Steggerda, J. Mullendore, S. Davis, L. Small and B. Duvall (all at the University of Maryland except L.S. Cutler, who is at Hewlett-Packard) during the period from May, 1975 through January, 1976 with the support of the U.S. Navy,[37] especially the Time Services Division of the U.S. Naval Observatory and its Director, G.M.R. Winkler. The Hewlett-Packard Model 5061 High Performance (Option 004) cesium atomic clocks used were provided by the observatory. They were modified to improve their performance under the guidance of L.S. Cutler,[38] who had been responsible for the original design at Hewlett-Packard. We were kindly lent two hydrogen masers for the ground clock set made by H. Peters by the Goddard Space Flight Center,[39] which also lent us the trailer used in the experiments. Three rubidium optically pumped frequency standards made by Efratom were included in each clock set.[40]

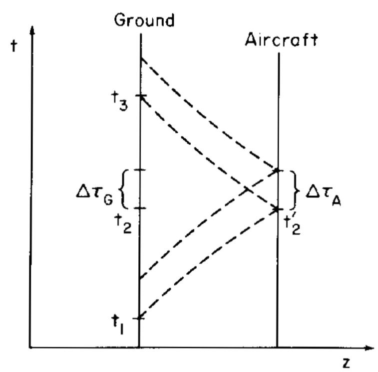
Theoretical Framework. Our experiments measured the difference in proper time recorded by the aircraft clock set, τ_A, and the proper time recorded by the ground clock set, τ_G. The prediction of general relativity is

$$\tau_A = \int_0^T (1 + \phi_A/c^2 - v_A^2/2c^2)\, dt \qquad (39)$$

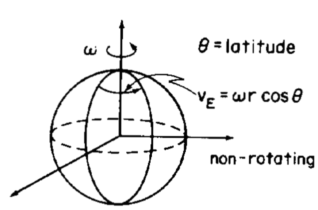
$$\tau_G = \int_0^T (1 + \phi_A/c^2 - v_G^2/2c^2)\, dt \qquad (40)$$

$$\tau_A - \tau_G = \int_0^T ((\phi_A - \phi_G)/c^2 - (v_A^2 - v_G^2)/2c^2)dt \qquad (41)$$

We used short pulses of laser light to carry out the comparison of clock readings between the ground and the airplane as prescribed by Einstein, equation (1) above. The appropriate spacetime diagram for the laser pulse time comparison is shown below.



The inertial system in which the integral (41) is to be evaluated is centered on the freely falling earth and is non-rotating with respect to distant matter, as shown in the following diagram.



The vector velocity $\vec{v}_A$ in the inertial system is given by

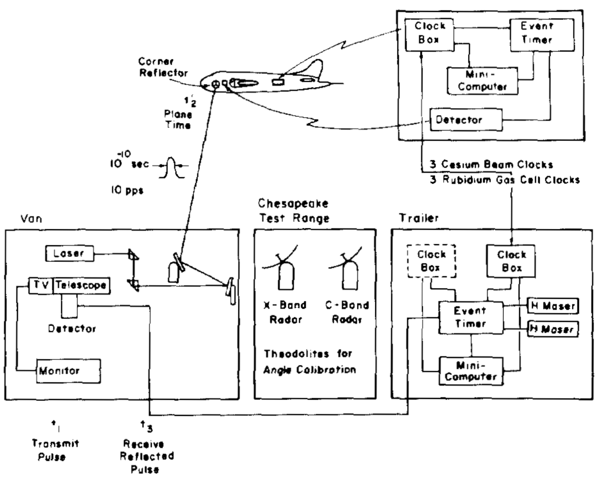
$$\vec{v}_A = \vec{v}_A^{\,*} + \vec{\omega} \times \vec{r} \qquad (42)$$

whose $\vec{v}_A^{\,*}$ is the velocity of the aircraft with respect to the surface of the earth and $\vec{\omega} \times \vec{r}$ is the surface velocity of the rotating earth whose angular velocity is $\vec{\omega}$. The radius vector to the location of the aircraft is $\vec{r}$. When $\vec{v}$ is squared to insert in equation (41), one obtains

$$v_A^2 = v_A^{*2} + \underline{2\,\vec{v}_A^{\,*} \cdot (\vec{\omega} \times \vec{r})} + (\vec{\omega} \times \vec{r})^2 \qquad (43)$$

The underlined term in the above equation is just twice the eastward component of $\vec{v}_A^{\,*}$ multiplied by the eastward surface velocity $v_E = \omega\, r \cos\theta$, where θ is the latitude. It is this term which leads to the asymmetry in the around the world flights described above, since it is positive for eastward flights and negative for westward flights. In our local flights, its integral was too small to be measured.

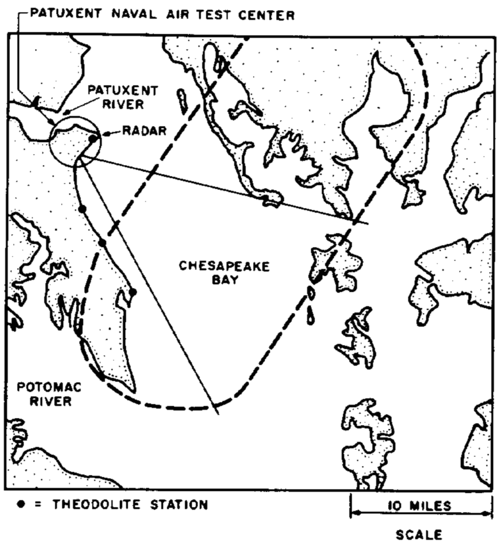
Airborne Equipment. A schematic diagram of the experiment is given below. The aircraft was a Navy P3C anti-submarine patrol plane, capable of flights of 15 to 16 hours duration at altitudes up to 35,000 feet. Figure 11 is a picture of our aircraft, P3C 912, in flight. On board the plane was an ensemble of three Hewlett-Packard cesium clocks



carefully packaged to provide a very well controlled environment along with an environmentally controlled set of three Efratom rubidium clocks, the entire collection of clocks and environmental package being called a "clock box." This is shown in Figure 12 in position on the aircraft. Also on board were an event timer, capable of measuring the epoch of an event with a precision of 0.1 nanosecond, a NOVA 2 minicomputer and associated LINC magnetic tape unit, a strip chart recorder for visual monitoring of clock phases and temperatures, a dry nitrogen tank for the pressure control system, and a non-environmentally controlled traveling clock. This equipment is shown in Figure 13. Intercomparison of the epochs of the zero-crossings of the 5 MHz clock outputs was made every 200 seconds. The reflection of laser light pulses from the aircraft was accomplished by placing an optical corner reflector of the type developed for the lunar laser ranging experiment[41] beside the forward observing window on the aircraft as shown in Figure 14. A photomultiplier equipped with neutral density filters and a 100 Angstrom band pass filter was housed as shown in Figure 15 so that it could be pointed by hand from inside the forward observing window toward the laser transmitter on the ground. The epoch of the received laser pulses on the plane was measured by the event timer in the proper time of the plane and stored in the computer where it could be read out and communicated to the ground by voice radio link.

Ground Equipment. On the ground at the Patuxent Naval Air Test Center, a trailer contained an identical clock box (Figure 16); event timer and NOVA 2 minicomputer with standard magnetic tape, disk, and tektronix graphics terminal for analyzing and displaying the data (Figure 17); and two hydrogen masers (Figure 18). Figure 19 shows the aircraft parked alongside the trailer for direct comparison by coaxial cable of the aircraft and ground clock sets before and after flights. Between flights in the absence of the aircraft, both clock boxes and the airborne electronics were housed in the trailer. On the five separate fifteen hour flights, one box was flown three times and the other twice. Figure 20 shows Dr. Williams and Dr. Reisse preparing to transfer a clock box to the plane, into which it had to be inserted through the narrow hatch shown in Figure 21. The installation of the electronics and clock box in the P3C aircraft typically required a day and a half.

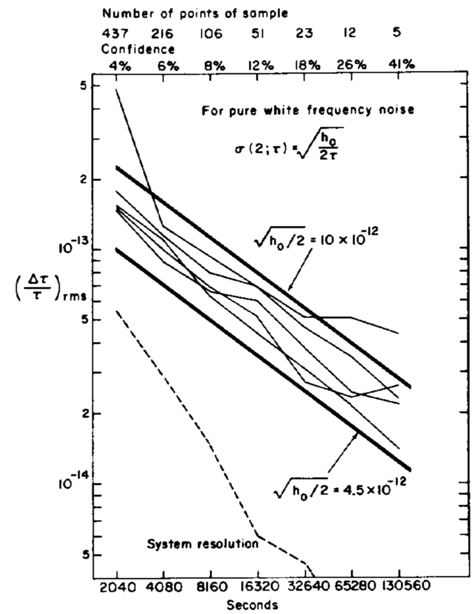
Laser Light Pulse Time Comparison. The laser transmitting and receiving equipment was located in the van at the corner of the hangar in Figure 19. The beam directing optics is pictured in Figure 22 and illustrated schematically in the above diagram. Underneath the laser was located a 7.5 inch telescope[42] which was used with a beam splitter both to detect the reflected laser light with a photomultiplier tube and to track the plane with closed circuit television. This equipment in the van is shown in Figure 23. The laser is a frequency doubled neodymium YAG system utilizing a mode-locked oscillator with pulse extraction and subsequent amplification, emitting a pulse 10 times a second with energy of 0.5 millijoules and duration of 100 picoseconds. The aircraft was flown mainly at night with its landing lights on to enhance the contrast for acquiring and tracking it visually. The flights approximated the path in a clockwise sense shown in the following map. The cone shows the acceptance angle for the laser light pulse to be reflected back by the corner reflector. The time comparisons were made on the near side of each circuit, with occasional gaps due to cloud cover or equipment difficulties. The aircraft was acquired optically



shortly before the laser time comparison by pointing in the direction communicated by the Chesapeake Test Range which tracked the aircraft continuously with radar. The laser beam had a divergence of about 0.5 milliradians, which illuminated about one-half the length of the aircraft at the slant range of about twelve miles. The required precision pointing was provided by Steve Davis working with coarse and fine Heath kit model airplane controllers setting elevation and azimuth rates for the beam directing optics. The controllers and closed circuit TV display are shown in Figure 24, while Figure 25 displays the landing lights as seen on the TV screen. When the laser pulse was hitting the corner reflector, the return flashes were seen on the screen and could also be seen by the eye if one stood next to the laser beam. At the plane, the laser light was very bright (but considerably below eye damage levels), actually casting shadows in the darkened interior of the plane. An inadequate view of the laser transmitter and runways as seen from the plane at twilight is shown in Figure 26. (This is one frame from a color motion picture film. It can be seen, along with other parts of the experiments, in the BBC TV program, "Einstein's Universe," presented on public television as part of the Einstein Centennial activities).

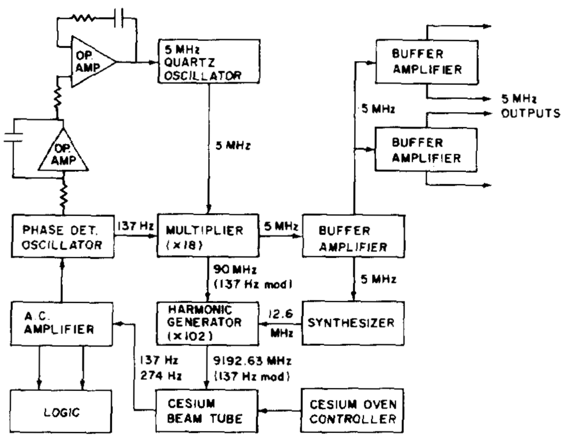
Radar Tracking. An essential part of the experiment was the continuous measurement of the aircraft altitude and velocity during flight by radars at the Chesapeake Test Range located about two miles from the ground clock set. At intervals during flights, the angular measurements of the radars were calibrated by optical theodolities. The radar data was used to compute the relativistic time integral, equation (41), with an accuracy considerably better than that of the atomic clock measurements themselves, so that the clock performance itself determined the accuracy of the comparison with general relativity. The Chesapeake Test Range and a closeup of one of the antennas are shown in Figures 27 and 28.

Clock Performance. Typical atomic clock stabilities of 2 to 3 parts in 10^{14} for an averagin time of one day were achieved with commercial Hewlett-Packard 5061 high performance standards by making several modifications to them and by maintaining a rigorously controlled environment. A plot of the experimentally measured Allan variance $\sigma\ (2;\ \tau)$ for five separate standards compared with the sixth is given below. The modifications consisted of a proprietary change in the beam tube



which is now standard on all HP5061 high performance units; the addition of an additional integration in the quartz crystal control loop to reduce frequency offset due to steps or ramps in the crystal frequency, (now available as an option from Hewlett-Packard); and the increase of the beam current by a factor of about two by raising the oven temperature (also available as an option from Hewlett-Packard), which increases short term frequency stability at the expense of tube lifetime. A simple block diagram of the standard is given below showing the extra integration and additional buffer amplification to aid in the intercomparison of the clocks. The modifications were carried out at Maryland with the supervision and active participation of L.S. Cutler. Figure 29 shows him tuning up the ensemble of clocks before they were inserted into the clock boxes.

BLOCK DIAGRAM OF HP 5061 CESIUM BEAM ATOMIC CLOCK



Clock Packaging for Environmental Control. Careful environmental control was maintained to keep small any changes in temperature, pressure, magnetic fields, or supply voltages. Isolation from vibrations and shocks was provided. These controls were accomplished by mounting each set of three clocks in a clock box, shown opened in Figure 30. The objective was to control the environment so that any changes would affect the clock stability by less than 10^{-14}. The clocks were mounted vertically in individual magnetic shields of 60 mil thick Mo-Permalloy. After the magnetic lid was put on (See Figure 31), individual clocks were degaussed. To remove heat from the clocks, and to control their temperature, air was circulated by an individual fan through each of the magnetic shields, entering at the top through the hoses shown in Figures 31 and 32 after passing a heater controlled by feedback from a distributed array of thermistors within the magnetic shield. The air exited a shield through holes at its bottom and flowed along the interior bottom of the pressure controlled aluminum clock box, losing heat to the outside through conduction. Variable speed fans controlled the flow of external air over the bottom of the clock box, the rate being controlled automatically by the external temperature. (We are indebted to Dr. J.P. Richard of the University of Maryland for this and other advice concerning the temperature control). The temperature at individual points in the clocks was kept constant to about 0.060°K. The temperature of the air changed by about 10° in flowing through a magnetic shield.

The clock box was nearly hermetically sealed when the lid was attached and either dry nitrogen or dry air was fed to it from a Granville-Phillips feedback controlled value to keep the pressure constant to less than 1 Torr at a value slightly above sea level atmospheric pressure. To allow for possible decompression of the aircraft cabin pressure at high altitudes, the clock box was made very strong with walls of ½ inch thick aluminum. This also isolated the clocks from acoustic noise.

The isolation against vibration and shock was achieved by mounting the box on four pneumatic Barry mounts which gave a resonant frequency of about 3 Hz. Almost critical damping was achieved by using expansion cylinders following an adjustable orifice, which led to an increase of isolation of 12 db per octave. The characteristic frequencies of the aircraft were around 80 Hz. Sway of the box in the aircraft was restrained by a cushioned retainer ring acting on a vertical post extended upward from the box. Strong flexible cables were fastened around the pneumatic isolaters to restrain the 1000 lb. box in the event of an accident.

The clock box lid carried the voltage regulation and pressure control equipment as well as the environmental control box for the Efratom rubidium clocks, which can be seen on the left in Figure 33. Similar control procedures were followed for these clocks, but they proved much more susceptible than the cesium standards to the shocks of landing and take-off, those events producing rate changes. For this reason, the rubidium data is not very useful for relativity measurements and will not be presented here.

Significant Features. Some of the significant features of the experiments are listed below:

*Ensembles of Clocks on Ground and in Aircraft

Clocks in each ensemble intercompared among themselves every 200 seconds.

*Careful Environmental Control

- Temperature stability $\Delta T < 0.060^{\circ}K$ (rms)
- Pressure stability $\Delta P < 1$ Torr
- Magnetic shielding
- Vibration and shock isolation
- Voltage regulation

Effect on clock stability < 10^{-14}
No need for systematic corrections

*Return of Clocks for Post Flight Comparison

No rate changes observed for cesium standards beyond statistical expectation for stationary clocks.

*Repeated Measurements

- Clock Box 1: 3 flights
- Clock Box 2: 2 flights
- 5 Test flights of ∿2 hours duration each to study and improve performance of experiment.

*Laser Light Pulse Time Comparison During Flights

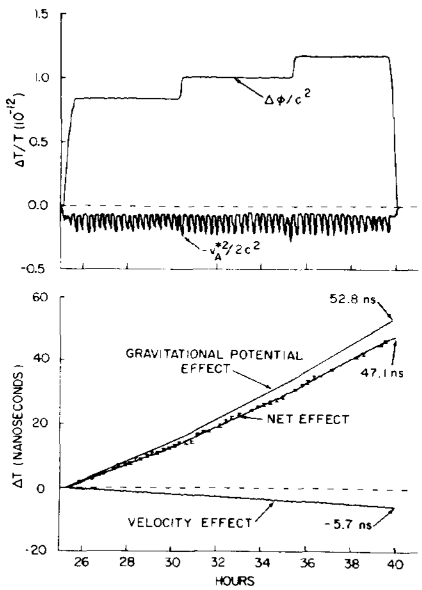
- No Doppler Effect Complications
- Technique of value for future space experiments since optical pulses are little affected by the ionosphere and solar corona.
- First realization of Einstein's 1905 prescription for comparing separated

clocks with light pulses.

*Flying Clocks Experience $\sim$ 1 g of Most of Time

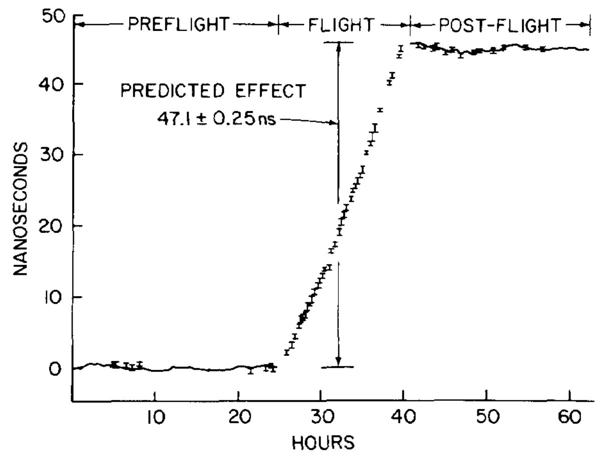
- -No relaxation of stresses as occurs in free fall.
- -Periods of steady acceleration limited to take-off and landing.
- -Plane's rotations made slowly to avoid Coriolis effects on cesium beams.

Experimental Results. Although much detailed data exists from the five 15 hour flights, only a representative sampling can be given here, which is taken from the flight of November 22, 1975. Shown below is a plot of the predicted effect of gravitational potential and of velocity on the rate of the aircraft clock set with respect to the ground clock set calculated from the radar tracking data.

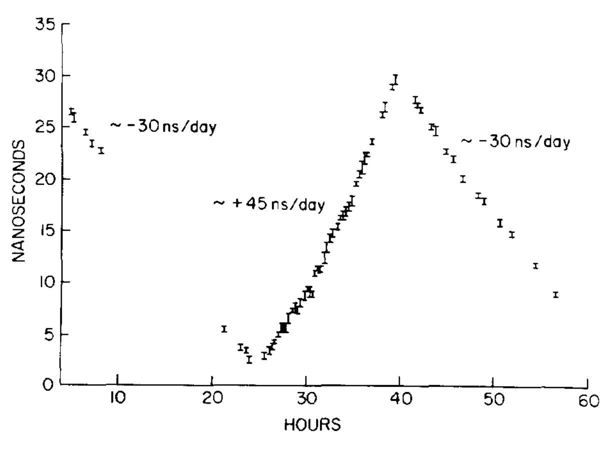


The upper curve is a plot of $(\phi_A - \phi_G)/c^2$ which is seen to average about 10^{-12}. The steps in this quantity are caused by the need of the aircraft to remain at 25,000 feet for 5 hours while burning off fuel to enable it to climb to 30,000 feet for another 5 hours, before being light enough to climb to 35,000 feet for the final 5 hours. The lower curve is a plot of $V_A{*}^2/2c^2$, the velocity effect due to motion of the plane with respect to the ground at the Chesapeake Test Range. The oscillations are due to the effect of winds as the plane circles. The average value of this effect is seen to be about 10^{-13}, corresponding to an average speed of 138 m/s. The second and third terms of equation (43) are not plotted since $\vec{V}_A{*} \cdot (\vec{\omega} \times \vec{r})$ will integrate to zero and $(\vec{\omega} \times \vec{r})^2$ will be nearly cancelled by a similar term for the ground clocks. The lower plots are the integrals of the upper curves over the 15 hour flight, showing the time difference $\Delta\tau = \tau_A - \tau_G$ as it was predicted to develop during the flight for the gravitational and velocity effects separately and for the net effect. The plotted points with error bars (± $\sim$ 0.3 ns) are the laser light pulse comparisons.

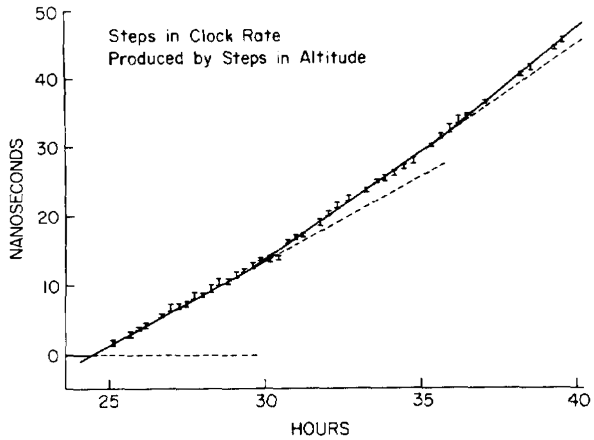
The direct comparison measurements of the 5 MHz clock phases before and after the flight are shown in the following plot along with the laser pulse comparisons which were made when the plane was on the ground as well as in the air. The solid lines are the comparison of the "paper clocks,"



that is the averages of the three cesium clocks in each clock set. The irregular form is due to the intrinsic clock time fluctuations. In this plot, the difference in rates between the clock sets established before the flight has been subtracted out. The actual rates are shown in the following plot of the laser time comparison.

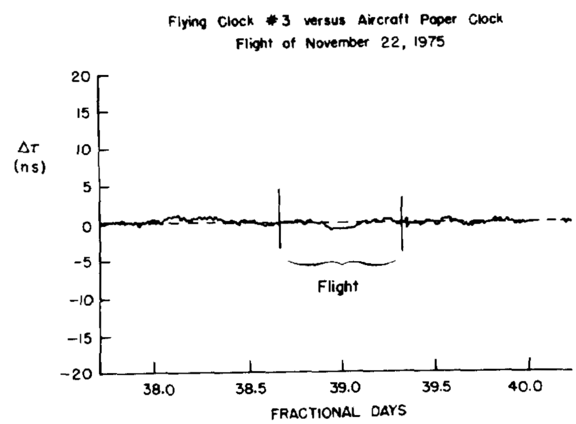


The gap in the preflight data was caused by the need of the laser operator to sleep before the flight. The effect of the steps in altitude on the flying clock rates can be seen by plotting the laser comparisons during flight in an expanded scale as is done below. The only change the clocks



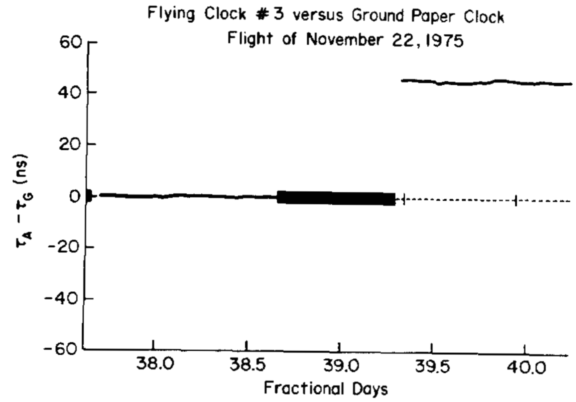
experienced was the change in gravitational potential.

It is also of interest that the flying clocks continued to behave with respect to one another exactly the same during the flights as before and after. This is illustrated by the following plot of the time of flying clock #3 with respect to the average of the three flying clocks. Similar data exists for each clock on each flight. When compared to the ground paper clock, however, flying

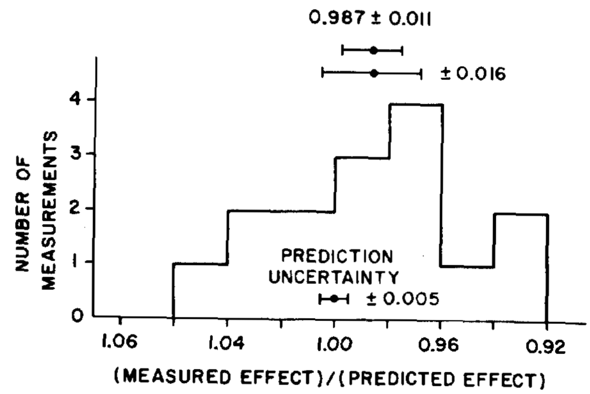


clock #3 showed a step as seen in the next plot. (The black marks fusing into a thick bar are recorded when a data point is absent for the direct comparison of the 5 MHz phases, as was the case during the flight). The step in $\tau_A - \tau_G$ is, of course, the result of the relativistic effects during the flight.

If one treats each cesium clock on each of the five flights as an individual measurement,



there is some scatter. Below is plotted a histogram for the ratio of the measured time difference to that calculated from General Relativity using the radar data. The predictions had an uncertainty no



more than ± 0.5%. The formal standard deviation of the mean was 0.011. To allow for possible systematic efforts, we are currently estimating (conservatively) an uncertainty of ± 0.016. The result is:

$$\frac{\text{Measured Effect}}{\text{Calculated Effect (General Relativity)}} = 0.987 \pm 0.016$$

The measured behavior of macroscopic clocks in aircraft flights - a "human scale" type of situation - exhibits the remarkable properties of Einsteinian time within the accuracy of measurement of about 1%!

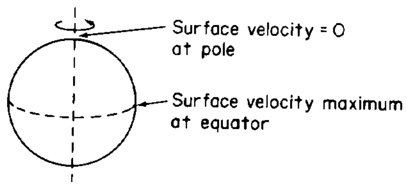
<u>Atomic Clocks Carried in an Aircraft on Global Flights With Inertial Navigation and Radar Altimetry</u>.

<u>Participants</u>. The flights were conducted in June and July, 1977 by the same group (from the University of Maryland and Hewlett-Packard) which conducted the local flights described above with similar equipment.[43] The Air Force joined the Navy in supporting the experiments by

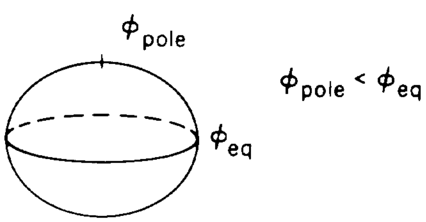
providing a C141 long range transport aircraft from the Wright Patterson Air Force Base, assistance in reconfiguring the equipment, and ground support facilities at the Andrews Air Force Base in Maryland near Washington, D.C.

Purpose. The purpose of the flights was to study experimentally the implications of General Relativity for the elapse of proper time at different latitudes on the spinning earth, freely falling towards the sun as it moves in its orbit. Clear understanding of these implications is of great practical importance for worldwide time keeping, and will be discussed later. Clarification of the concepts by means of simple experiments aids in the development of physical intuition.

Null Change of Proper Time with Latitude. On the spinning earth, general relativity predicts a null effect on proper time as one moves a clock from the equator to the pole along the mean ocean surface. This was discussed earlier in the sections on theory in relation to Einstein's discovery of the effect of gravity on time. His 1905 prediction that a clock at the equator would run slow with respect to one at the pole ignores the influence of gravitational potential since he did not discover it until 1907. If the earth were a homogeneous perfect sphere as shown, the gravitational potential would be the same everywhere on the earth's



surface and the 1905 prediction would be correct. However, the earth has the shape, to first order, of an oblate ellipsoid due to its spin, as shown in exaggerated form in the following sketch. The gravitational potential ϕ_{pole}, at the pole is less



than that of the equator, ϕ_{eq}, since the pole is closer to the center of the earth. The surface of the oceans of the earth is determined by the "geopotential," $\phi - V_s^2/2$, where V_s is the surface speed of the rotating earth. $V_s^2/2$ is the "centrifugal potential, and ϕ is the gravitational potential, both of which change with latitude such that the combination has a constant value independent of latitude,

$$\phi - \frac{V_s^2}{2} = \text{constant} \tag{44}$$

along the ocean surface. From the earlier discussion of the theory, it will be recalled that the relation between a proper time increment $\Delta\tau$ and a coordinate time increment Δt is

$$\Delta\tau = \left(1 + \frac{\phi}{c^2} - \frac{V^2}{2c^2}\right) \Delta t \tag{45}$$

for the weak gravity and slow velocities which exist at the earth's surface. Thus, along a surface of constant geopotential, the relation of proper time to coordinate time is

$$\Delta\tau = \left(1 + \frac{\text{constant}}{c^2}\right) \Delta t \tag{46}$$

and all standard clocks at mean sea level are predicted to run at the same rate independent of latitude.

Experimental Results for Flight to Thule. To check this null prediction, a clock box containing three cesium standards was transported in an Air Force C141 from Washington, D.C. to Thule Air Force Base in Greenland as shown in the polar photograph of a globe in Figure 34, this being the largest conveniently accessible latitude change in one hemisphere. The latitude of Thule is 76° 32' and that of Washington is 38° 49'. After a number of local test flights with the reconfigured equipment in the C141 aircraft, the flight to Thule was undertaken on June 23, 1977, after several days of problem-free comparison with the ground clocks, as a combination of long test flight and global measurement. The plane was kept on the ground at Thule for four days and returned on June 27. Although the radar altimeter failed about 2 hours before arrival at Thule and had to be replaced by one flown in on a regular Air Force flight, the experiment was generally successful. Using the inertial navigation data and radar and pressure altimeters to evaluate the relativistic integral, equation (41), and making direct electrical comparisons of the aircraft and ground clock sets before and after the flight as had been done for the local flights from the Patuxent Naval Air Test Center, but with pre and post flight periods of several days, the following comparison was obtained.

Measured: $\tau_A - \tau_G = 38 \text{ ns} \pm 5 \text{ ns}$

Calculated: $\tau_A - \tau_G = 35 \text{ ns} \pm 5 \text{ ns}$

The time difference measured agrees within errors with that predicted for the flights to and from Thule. There is no evidence for any anomalous latitude effect. The "Einstein Error" of 1905 - not including the gravitational effects - would have predicted a time difference of an additional 56 ns per day, or a total of 224 ns for the four day dwell time! Because the aircraft had to be used for other purposes starting August 1, it was not possible to repeat the Thule flight or to make flights from Washington to Panama to check further the null prediction. It was deemed more important to use the remaining time to transport the clocks to Christ Church, New Zealand.

Reconfigured Equipment for C141 Aircraft. Before describing the purpose and results of the northern to southern hemisphere flights, the reconfigured equipment and the ground base at the Andrews Air Force Base will be briefly discussed. Figure 35 shows a front view of the equipment being assembled on an aluminum frame in the shop of the University of Maryland.[44] The clock box, seen on the left, was rewired to meet rigorous Air Force requirements. It was also mounted on improved pneumatic supports at the height of its center of mass to reduce the sway encountered with the former bottom mounting. The minicomputer, event timer, tape recorder, chart recorder, and other electronics were mounted in an Air Force furnished rack which included some vibration and shock isolation between the inner structure and the outer structure which was bolted to the main frame.

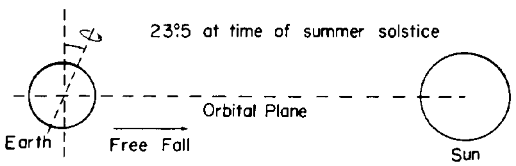
Figure 36 is a rear view of the frame and equipment. A special regulated power supply capable of working from either 60Hz or 400Hz was constructed to power the clocks and other equipment. It included enough Sears Die Hard batteries in a special vented container to operate the clocks for at least 12 hours in the event of failure of other sources. This apparatus is seen on the left, being worked on by technician Lyndon Small.

The entire frame was surrounded by double wall plywood panels containing insulation and was mounted on a standard Air Force 7 foot by 9 foot cargo pallet. Figure 37 is a picture of the complete assembly in a prefabricated garage structure built at the Andrews Air Force Base to contain the equipment between flights. The hoses on the back panel circulate temperature controlled air into and out of the large insulated enclosure. Two Sears window air conditioners provide cooling. Heaters in the flat plenum on the left were used for fine temperature control which could be maintained to a few degrees. This enclosure solved one of the major problems -- the large variation with time of the temperature within the aircraft during flights, and on the ground. The mounting on the cargo pallet allowed the equipment to be placed on the C141 or removed from it in about 10 minutes compared with about one and a half days for the earlier experiments with the Navy P3C aircraft. Figure 38 shows the C141, garage, trailer housing the ground clocks, and van containing the ground computer equipment. Figure 39 shows the tail assembly of the C141 with the petal doors which can open to receive the pallet with its clock equipment. The first transfer of the equipment (without its front insulating panel) from the garage to the plane is shown in Figures 40 through 44. The installed equipment with Len Cutler standing in front is seen in Figure 45. During an actual flight, the equipment is shown from the front in Figure 46 and from the rear in Figure 47. Enough seats were installed to accommodate the eight University of Maryland people and fourteen Air Force personnel who went on each flight. In addition to the crew for operating the plane, there were technicians to operate and maintain the gasoline powered portable electrical generators, portable air conditioners, and portable heaters needed to power the plane and provide temperature control of its interior during dwell times on the ground at the remote sites and at Andrews Air Force Base before and after flights. The transportation of this equipment, some of it in duplicate, required the large storage capacity of the aircraft. It was essential in maintaining continuous operation of the clocks in an adequate environment in the interior of the plane at the remote sites, and during the 12 hour layovers in Hawaii on Christ Church flights.

Two Carousel IV inertial navigation systems, a radar altimeter, and a pressure altimeter were installed on the aircraft to provide accurate measurements of latitude and longitude, velocity with respect to the ground, and altitude above the ground. This information was recorded automatically in digital form every 0.6 second on magnetic tape recorders. In addition, there was provision for manual readout which was logged every 15 minutes. This log was used with a Hewlett-Packard 97 calculator to compute a running integral of the difference between the proper time on the aircraft and the proper time at the Andrews Air Force Base as the flights proceeded. The recording and readout equipment is seen in Figure 48. At Thule it was possible to place the aircraft in a hangar, shown in Figure 49.

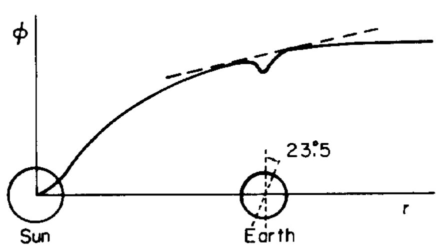
The midnight sun was very high in the sky at Thule since we were there only a few days after the summer solstice. Figure 50 showing the C141 which brought our replacement radar altimeter was taken just before midnight. The relatively short shadows cast by the sun at midnight are shown in Figure 51. The $23^{\circ}.5$ tilt of the earth's spin axis which causes this summer solstice on the 21st of June each year in the northern hemisphere made possible another experiment which required flying the clock from the northern to the southern hemisphere.

Does the Gravitational Potential of the Sun Affect Proper Time on the Freely Falling Earth? The following diagram shows that at the time of the summer solstice the North Pole and points in



the northern hemisphere are closer to the sun on the average (due to the spin of the earth) than the South Pole and points in the southern hemisphere. Since the earth is falling freely towards the sun as it moves in its yearly orbit around the sun, the nearly fixed direction of the spin axis affords the opportunity of conducting experiments in a freely falling laboratory -- the earth as Einstein's freely falling elevator. At the time of the Summer Solstice, the northern hemisphere is the "floor" and the southern hemisphere is the "ceiling," with the positions reversed at the Winter Solstice.

Recalling the earlier discussion about Einstein's Principle of Equivalence, one can ask: is the gravitational field of the sun transformed away for experiments on the freely falling earth? In particular, will clocks on the "floor" run at the same rate as clocks on the "ceiling" even though they are at different distances from the sun? Here we are taking for granted the effect of gravitational potential difference on clock rates as established by other experiments. The question can be answered experimentally by transporting clocks from the northern to the southern hemisphere near the epoch of a solstice, letting them dwell for a while, and then returning them for comparison with the stay-at-home clocks. From the theoretical point of view, the Principle of Equivalence seems to provide a clear answer: on the freely falling earth (strictly, the earth-moon system, but we ignore this complication since its effects are small) the gravitational effects of the sun are not experienced, to first order, just as in the freely falling Skylab in orbit about the earth the astronauts experienced no effects of the gravity field of the earth, to first order. Mathematically, this means that in the plot of gravitational potential ϕ as a function of distance from the sun, as shown in the sketch below, the slope, or linear



term in the Taylor's Series expansion of ϕ about the center of the earth is subtracted away, leaving only the second order terms.[31] These terms produce the tides, but the difference in ϕ which they yield across the earth's diameter would produce a rate difference in proper times of only 7×10^{-17} or 0.62 picoseconds per day. However, the linear term, if it affected phenomena on earth, would produce a day to night shift in clock rates of about 8×10^{-13} or about 75 ns per day. This question was studied carefully by Professor Banesh Hoffmann (author of the biography of Einstein recommended in the first section of this paper[4]) in 1957.[45] He predicted that no such effect would be measured in the reference frame of the earth, but called for experiments to check the prediction when clocks of sufficient accuracy became available.

In 1976 the question was examined again by Professor Roman Sexl[46] who was seeking an explanation for an erroneous report of a dependence of atomic clock rates on latitude.[47] He concluded that there should be a seasonal effect caused by the tilt in the earth's spin axis, described by the equation

$$\frac{d\tau_1}{dt} - \frac{d\tau_2}{dt} = 14.8\{\sin\Theta_2 - \sin\Theta_1\} \cdot \cos\left[\frac{t - 21\ \text{June}}{365}\right]\ \text{ns/day} \tag{47}$$

where τ_1 and τ_2 are the proper times at latitudes Θ_1 and Θ_2 (southern latitudes being taken as negative). This equation results from the incorrect retention of the linear term in the expansion of the sun's potential as just discussed. It would be correct for observations conducted from a frame of reference attached to the sun, but is wrong for observations carried out on the earth. Professor Sexl now acknowledges his error.[48]

Experimental Results for Flights to Christ Church. Figure 52 shows on a globe tilted at 23°.5 the path taken by our C141 in transporting the clocks to Christ Church, New Zealand (latitude -43°29') from Washington, D.C. (latitude 38°48') and the different average distances of these locations from the sun. The first trip was made from July 10 to July 17 and a second trip from July 23 to July 30. Three days were required for the travel out and back, including ∿12 hour stopovers in Hawaii each way and a 2 hour stopover at the Travis Air Force Base in California on the westward journey. This allowed a dwell time in Christ Church of four days. Figure 53 shows the aircraft on the ground at the Operation Deep Freeze Antarctic support base in Christ Church surrounded by some of the portable electrical generating and heating equipment (it was winter in New Zealand) it had carried with it.

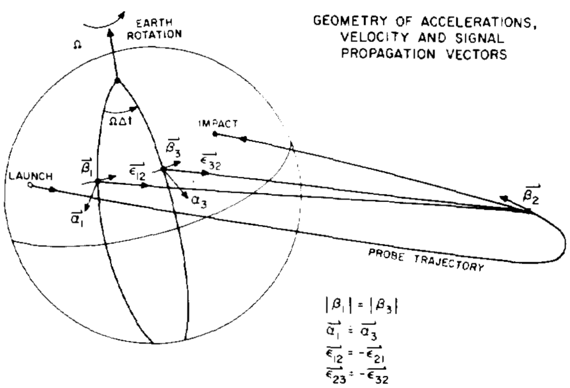
The data for the time differences $\tau_A - \tau_G$ are displayed in the following table:

	Measured Value (ns)	Calculated Value (ns)	Alleged Effect of Sun (ns)
Flight 1	115 ± 8	129 ± 5	80 ± 5
Flight 2	131 ± 8	118 ± 5	70 ± 5

The data is still being analyzed to correct the readings of the radar altimeter over the varying topography of the continental U.S. to give altitude above the ellipsoid rather than above the local topography, but the changes in the above numbers will not be large. There is no evidence, with a detection sensitivity of about 10% when the measured and calculated effects are compared, of the alleged effect of the sun.

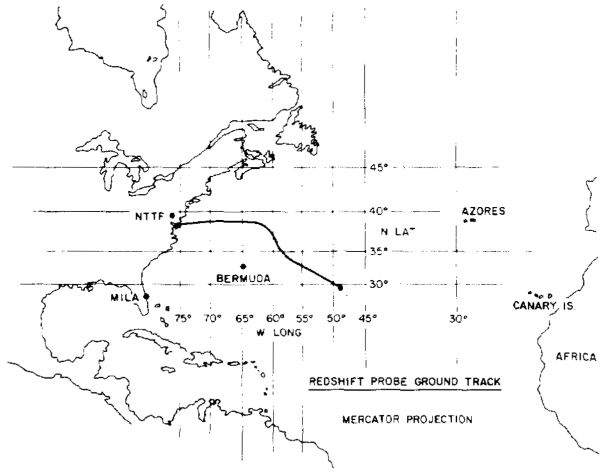
Hydrogen Maser Carried on Suborbital Rocket Flight With Doppler Cancellation Tracking

In June 1976, a hydrogen maser was carried as payload in a suborbital flight by a Scout rocket launched from the Wallops Test Center of NASA in an experiment designed and conducted by R.F.C. Vessot, M. Levine and others of the Harvard/Smithsonian Center for Astrophysics.[49] The purpose was to measure with high accuracy by ground tracking the large change in frequency of the maser caused by the large change in gravitational potential as it ascended to an altitude of about one and a half earth radii above the earth's surface and fell back into the Atlantic Ocean in a flight lasting just under two hours. A sketch of the trajectory is given below. The ground path of the flight is shown in the following map. Ground tracking with microwave frequencies was carried out from stations

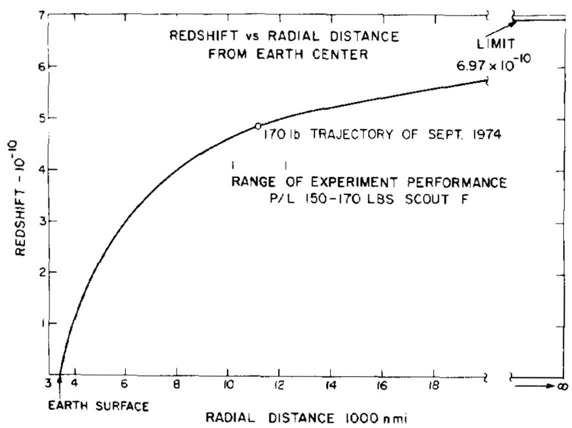


at Wallops Island, Bermuda and Florida equipped with hydrogen masers. In terms of relativistic frequency changes, it is readily shown that they are expressible as the integrand of equation (41)

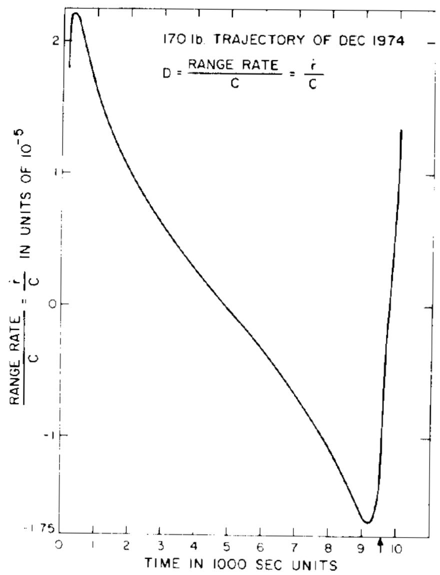
$$\frac{f_{Probe} - f_{Ground}}{f_{Ground}} = \frac{\Delta f}{f} = \frac{\phi_{Probe} - \phi_{Ground}}{c^2} - \frac{v^2_{Probe} - v^2_{Ground}}{2c^2} \qquad (48)$$



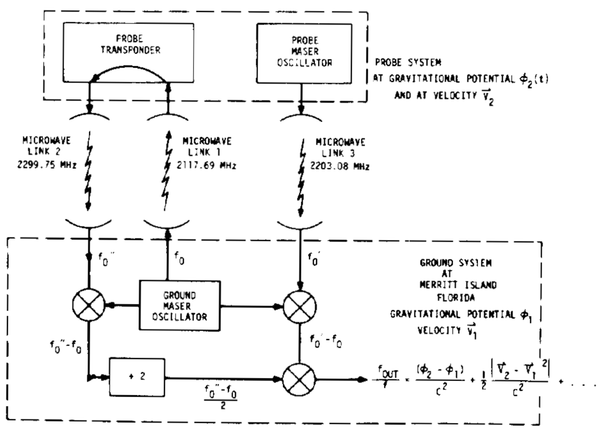
In this case, the gravitational potential difference causes the frequency of the probe maser to be shifted to higher values than the reference masers on the ground -- a violet shift rather than a red shift, but it is still convenient to speak of the effect as "redshift." The value of the gravitational shift as a function of height above the earth is plotted below. The received microwave frequency



will be strongly affected by the ordinary Doppler effect which is just the rate of change of slant range divided by the speed of light. For a typical planned trajectory, the Doppler shift in frequency could be as high as 2 x 10^{-5} as shown in the following plot. Since the gravitational relativistic effect would be about 4 x 10^{-10} and the expected stability of the hydrogen maser was hoped to be $\sim 10^{-14}$, extraordinary measures had to be taken to measure or cancel the Doppler shift. Also, the large and variable effects of the earth's ionosphere causing phase shifts in the propagated microwave signals had to be considered.

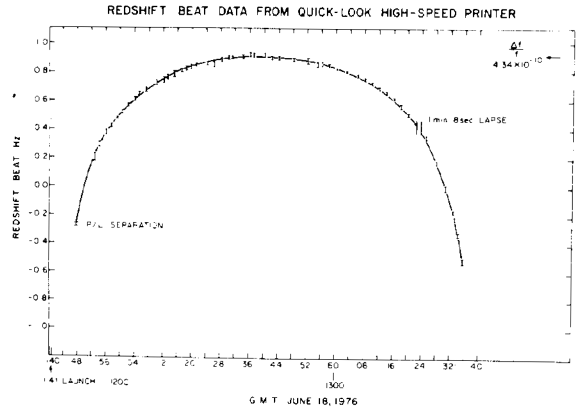


A very ingeneous arrangement of three microwave links at different frequencies as shown in the simplified block diagram below solved these problems.



REDSHIFT EXPERIMENT DOPPLER CANCELLING SYSTEM

It was possible to achieve real time cancellation so that the redshift was evident during the flight as shown by the following quick look beat frequency data. The break in the data just before the end of the flight is due to tracking station problems and interrupted the phase continuity. It could not subsequently be recovered, so the remaining data is not useful for analysis. The combination of potential and velocity effects produced zero beats during both ascent and descent. The ascent zero beat data is shown in Figure 54. The rotation and nutation of the spinning payload can be clearly distinguished. Figure 55 shows the environmentally controlled hydrogen maser package which was flown



(and not recovered!) being held by its makers, Bob Vessot (on the right) and Marty Levine. Figure 56 is a picture of the rocket during launch.

Careful analysis of the ground tracking data to determine the actual rocket trajectory and comparison of the measured frequency shift as a function of time with that predicted by general relativity has yielded the following value[50] for the gravitational frequency shift (the velocity effects were assumed given)

$$\frac{\Delta f}{f} = \{1 + (5 \pm 126) \times 10^{-6}\} \frac{\Delta\phi}{c^2} \qquad (49)$$

This is the most accurate measurement of the relativistic effects on frequency. A plot of the residuals at the early stage of the analysis is given in Figure 57.

Other Atomic Clock Measurements

Mountain to Valley Experiments. The first such measurement was made in 1975 in Italy between Torino and a cosmic ray laboratory at Plateau Rosa 3250 m higher by L. Briatore and S. Leschiutta [51] using one cesium beam clock at each location. Comparison was achieved by receiving the same TV signal at each site and by direct comparison before and after the 66 day dwell time at Plateau Rosa. Although no stringent environmental control was attempted, nor were systematic corrections made for environmental changes, a 15% measurement was achieved, agreeing with the general relativistic calculation within that uncertainty.

The second measurement was made in Japan in June and July, 1977 by S. Iijima and K. Fujiwara[52] of the Tokyo Astronomical Observatory. One Hewlett-Packard 5061 High Performance standard was transported from the Tokyo Observatory (altitude 58 m above sea level) to the Norikura Corona Station (altitude 2876 m above sea level) on two successive occasions and left to operate for one week. Before and after those trips, one week comparisons were made with a similar clock at the observatory. Systematic corrections were made for the effect of different environmental conditions at the different sites on the rate of the clock. They also paid close attention to the placement of the clocks in the local magnetic field and took great pains to keep environmental control during the 8 hour transport time in a cushioned and air conditioned van. Their results were

$$\frac{\text{Measured Effect}}{\text{Calculated Effect}} = 0.94 \pm 0.05, \tag{50}$$

a verification at the 5% level.

These experiments measured purely the effect of gravitational potential difference.

Comparison of Absolute Frequency Determinations. Since the early 1970's, the National Bureau of Standards in Boulder, Colorado, at an altitude of about 6000 feet above sea level, has been including the gravitational effect $\Delta\phi/c^2 \sim 2 \times 10^{-13}$ in the determination of the absolute cesium frequency with its laboratory beam tubes. Other standards laboratories are at much lower altitudes so the effect is not yet significant in terms of achievable accuracies for them. However, the international definition of the second is based on the value of the cesium ground state hyperfine transition at sea level, so the gravitational shift of 1.09×10^{-16} per meter must be included in any absolute measurement. Inclusion of the effect in the NBS measurements does result in smaller differences in current international comparisons.

Practical Applications

Global Timekeeping

At the level of 0.1 microsecond, the relativistic effects of clock transport by aircraft are quite significant. For example, in the global measurements involving trips to Christ Church, New Zealand from Washington, D.C. and back, the relativistic time difference produced was ∿120 nanoseconds. It is interesting to give a breakdown of the calculated relativistic effect for each leg:

Washington to California	:	+29 ns	E→W
California to Hawaii	:	+31 ns	E→W
Hawaii to New Zealand	:	+52 ns	E→W
New Zealand to Hawaii	:	+16 ns	W→E
Hawaii to Washington	:	− 6 ns	W→E

The asymmetry between E→W and W→E is very apparent. It is clear that even for short flights, the effects are important at the ten's of nanoseconds level. The U.S. Naval Observatory is now using an algorithm to estimate these effects for its transportable clock trips.

NAVSTAR/Global Positioning System[53]

This is a new navigation system under development by the United States Department of Defense. It is shown in artist's conception, together with some facts about it in Figures 58 and 59. There will be 24 satellites placed in 12 hour period circular orbits, 8 in each of 3 orbital planes inclined at 63° to the equator and equally spaced around it. Each of these satellites will carry a very stable atomic clock (with several spares) and will transmit a pseudo-random noise code with a bit repetition frequency controlled by the atomic clock. Information about the orbit of the satellite is also transmitted. User equipment will receive signals from four or more GPS satellites simultaneously, or sequentially, locking on to the transmitted code by shifting a local clock which steps the same pseudo-random noise code in the receiver. Large scale integrated solid state electronic circuits in the receiver package can then calculate the position, velocity, and time for any user.

The stable atomic clocks in orbit are the heart of the system. In order for all of them to remain synchronized with each other and with the ground clocks at the U.S. Naval Observatory which sets time worldwide for the Defense Department, the effects of general relativity must be included. When placed in such an orbit, a standard clock will run fast with respect to an identical one on the ground by 44,000 nanoseconds per day! This effect was first measured with the NTS-2 satellite carrying a cesium atomic clock.[54] The clocks must be adjusted to run slow by that amount before being placed in orbit in order to keep in synchronization with atomic clocks on the earth's surface. Furthermore, periodic change in distance from the center of the earth for a satellite in a slightly elliptic orbit can lead to significant effects. For an eccentricity of 0.005, the peak to peak time difference will be 24 nanoseconds. This translates into a range uncertainty of 24 feet which is very significant in a system whose present design goal is 10 meters, and must be included in the system operation.

The fact that standard clock rates are constant at mean sea level on the ellipsoidal surface of the spinning earth as discussed earlier allows the establishment of a coordinate time for the Global Positioning System which can be made the same as Universal Time on the surface of the earth. The standard atomic clocks in orbit will keep this GPS time when they are offset by the ∿44,000 ns/day discussed above and corrected for the relativistic effects of the elliptic orbit. Stationary standard clocks on the ground must have their rates compensated, depending on their distance above or below the mean ocean surface ellipsoid by $\pm 1.09 \times 10^{-16}$ per meter in order to keep GPS coordinate time.

Significance

The two applications discussed briefly above are among the first non-scientific uses of Einsteinian Time. That the remarkable behavior of clocks as predicted by General Relativity is now required to be included in practical applications is an intellectual milestone, made possible by the extraordinary stability of modern atomic clocks.

References and Notes

1. Quoted in Hilbert, by Constance Reid. Published by Springer-Verlag: New York, Heidelberg, Berlin, 1970, page 105.

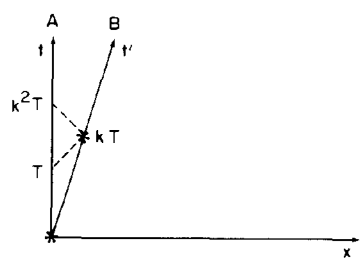
2. Martin Klein, "Einstein and the Academic Establishment," in Einstein, A Centenary Volume, edited by A.P. French, Harvard University Press, Cambridge, Massachusetts, 1979, page 209. (Figure 6 is taken from this essay)

3. Albert Einstein, "On Education" in Out of My Later Years, New York Philosophical Library, 1950. Reprinted in the commemorative volume cited in reference 2, page 315.

4. Banesh Hoffmann, Albert Einstein, Creator and Rebel. The Viking Press, New York, 1972. Paperback edition, New American Library, New York, London, and Scarborough, Ontario, 1973.

5. Hermann Bondi, Relativity and Common Sense, A New Approach to Einstein, Anchor Book Science Study Series, Doubleday and Company, Inc., Garden City, New York, 1964.

6. Consider the following spacetime diagram in which observer B is moving with constant velocity v with respect to observer A.



When B is at the same position as A, they set their identical clocks to read 0. At time T later A emits a light pulse which is received by B at time kT and immediately transmitted back to A who receives it at time $k(kT) = k^2T$ since the Doppler stretching has operated twice. By equations (1) and (2), the position of B as determined by A when B receives and reflects the pulse is

$$x = \frac{c}{2}(k^2 T - T) = \frac{c}{2}(k^2 - 1)\,T$$

and the time of this reception event by B according to A is

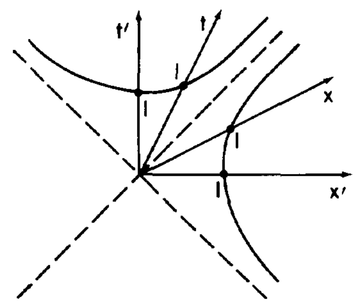
$$t = \frac{1}{2}(T + k^2 T) = \frac{1}{2}(k^2 + 1)\,T$$

The velocity $v = x/t = c\,(k^2 - 1)/(k^2 + 1)$ and the result in the text follows by solving for k.

7. The distances corresponding to the units on the time and space axes are determined by plotting branches of the rectangular hyperbolae

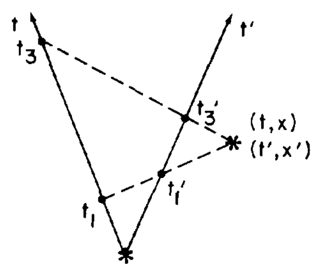
$$t^2 - x^2 = \pm 1$$

with respect to the light cone as shown in the following diagram:



Intersection of the time and space axes for an inertial observer yields the calibration points.

8. This is shown very easily with the "k-calculus" and the following diagram



The spatial axes have been suppressed. It is clear from equations (1) and (2) in the text that

$$t_1 = t - x/c \qquad t_1' = t' - x'/c$$

$$t_3 = t + x/c \qquad t_3' = t' + x'/c$$

It is also clear from the definition of the Doppler factor k, that

$$t_1' = kt_1$$

$$t_3 = kt_3'$$

These two equations may be written as

$$t - x/c = \frac{1}{k}(t' - x'/c)$$

$$t + x/c = k(t' + x'/c)$$

Multiplying these equations together and multiplying the result by c^2, one has the result stated as equation (4) in the text. If one solves this pair of simultaneous equations for t' and x' in terms of t and x, the Lorentz transformation is obtained.

9. New York Times, March 28, 1972; page 32. The first part of this excerpt is also very illuminating, so we reproduce it here:

"In the development of special relativity theory, a thought - not previously mentioned - concerning Faraday's work on electromagnetic induction played for me a leading role.

According to Faraday, when a magnet is in relative motion with respect to a conducting circuit, an electric current is induced in the latter. It is all the same whether the magnet moves or the conductor; only the relative motion counts, according to the Maxwell-Lorentz theory. However, the theoretical interpretation of the phenomenon in these two cases is quite different:

If it is the magnet that moves, there exists in space a magnetic field that changes with time and which, according to Maxwell, generates closed lines of electric force -- that is, a physically real electric field; this electric field sets in motion movable electric masses (that is, electrons) within the conductor.

However, if the magnet is at rest and the conducting circuit moves, no electric field is generated; the current arises in the conductor because the electric bodies being carried along with the conductor experience an electromotive force, as established hypothetically by Lorentz, on account of their (mechanically enforced) motion relative to the magnetic field.

The thought that one is dealing here with two fundamentally different cases was, for me, unbearable. The difference between these two cases could not be a real difference, but rather, in my conviction, could be only a difference in the choice of reference point. Judged from the magnet there certainly were no electric fields; judged from the conducting circuit there certainly was one. The existence of an electric field was therefore a relative one, depending on the state of motion of the coordinate system being used, and a kind of objective reality could be granted only to the electric and magnetic field together, quite apart from the state of relative motion of the observer or the coordinate system. The phenomenon of the electromagnetic induction forced me to postulate the (special) relativity principle."

10. P.G. Roll, R. Krotkov, and R.H. Dicke, "The Equivalence of Inertial and Passive Gravitational Mass," Ann. Phys (U.S.A.) 26, 442-517 (1964).

V.B. Braginsky and V.I. Panov, "Verification of the Equivalence of Inertial and Gravitational Mass," Zh. Eksp. & Teor. Fiz. 61, 873-879 (1971). English translation in Sov. Physics - JETP Lett. 10, 280-283 (1972).

11. C.F. Gauss, "Disquitiones Generales circa Superficies Curvas," Göttingen, 1827. There has just appeared a modern translation of "General Investigations of Curved Surfaces" in Astérisque 62: "150 Years After Gauss," by P. Dombrowski, published by the Société Mathématique de France, 1979.

12. C.W. Misner, K.S. Thorne, and J.A. Wheeler, Gravitation, W.H. Freeman and Company, San Francisco, 1973. Cover drawing and page 4.

13. Bertrand Russell, The ABC of Relativity, Third Revised Edition, New York, New American Library Mentor Book, 1959, page 80. (Original edition, 1925).

14. J. Evershed, Bulletin of the Kodaikanal Observatory, 36 (1914).

15. W. Pauli, Theory of Relativity. Pergamon Press; New York, London, Paris, Los Angeles. 1958, Page 153.

16. J. Brault, Bulletin of the American Physical Society, 8, page 28 (1963).

17. F. Roddier, Annals of Astrophysics, 28, page 463 (1965).

18. J.L. Snider, Physical Review Letters, 28, page 853 (1972).

19. J.L. Greenstein, J.B. Oke, and H.L. Shipman, Astrophysical Journal, 169, page 563 (1971).

J.L. Greenstein and V. Trimble, Astrophysical Journal, 149, p. 283 (1967).

20. A discussion of the problems involved is given by S. Weinberg, Gravitation and Cosmology, John Wiley and Sons, Inc., New York, London, Sydney, Toronto. 1972. Pages 79ff.

21. H.E. Ives and G.R. Stilwell, "An Experimental Study of the Rate of a Moving Atomic Clock," (I and II) Journal of the Optical Society of America, 28, p. 215 (1938); 31, page 369 (1941).

22. G. Otting, Physikalische Zeitschrift, 40, page 681 (1939).

23. H.I. Mandelberg and L. Witten, Journal of the Optical Society of America, 52, page 529 (1962).

24. J.J. Snyder and J.L. Hall, Laser Spectroscopy, Proceedings of the Second International Conference, 23-27 June, 1975, Mégève, France, Springer-Verlag: Berlin, Heidelberg, New York, 1975. Page 6.

25. S. N. Bagayev and V.P. Chebotayev, Pis 'ma Zh. Eksp. i Teor. Fiz., 16, page 614 (1972).

26. R.V. Pound and G.A. Rebka, Physical Review Letters, 3, page 439 (1959); 4, page 337 (1960).

27. R.V. Pound and J.L. Snider, Physical Review Letters, 13, page 539 (1964); Physical Review 13, 140, page 788 (1965). See also R.V. Pound, "Terrestrial Measurements of the Gravitational Red Shift," in Albert Einstein's Theory of General Relativity, edited by G. Tauber, Crown Publishers, Inc., New York, 1979. Pages 132ff.

28. H.J. Hay, J.P. Schiffer, T.E. Cranshaw, and P.A. Egelstaff, "Measurement of the Red Shift in an Accelerated System Using the Mössbauer Effect in Fe," Physical Review Letters, 4, page 165 (1960).

29. W. Kündig, "Measurement of the Transverse Doppler Effect in an Accelerated System," Physical Review, 129, page 2371 (1963).

30. K.C. Turner and H.A. Hill, "New Experimental Limit on Velocity Dependent Interactions of Clocks and Distant Matter," Physical Review B, page 134 (1964).

31. J.B. Thomas, "Reformulation of the Relativistic Conversion Between Coordinate Time and Atomic Time," Astronomical Journal, 80, No. 5, p. 405 (1975).

32. A.J. Greenberg, D.S. Ayres, A.M. Commach, R.W. Kenny, D.O. Cadwell, V.B. Elings, W.P. Hesse, and K.J. Morrison, "Charged Pion Lifetime and a Limit on a Fundamental Length," Physical Review Letters, 23, page 1267 (1969).

33. F.J.M. Farley, J. Bailey and E. Picasso, "Experimental Verifications of the Special Theory of Relativity," Nature, 217, page 17 (1968).

34. B. Rossi and D.B. Hall, "Variation of the Rate of Mesotrons with Momentum," Physical Review, 59, page 223 (1941).

35. J.C. Hafele and R.E. Keating, Science, 177, page 166 and page 168 (1972).

36. In addition to the main participants listed in the text, the following people participated from the University of Maryland: J.P. Richard and D.G. Currie advised on some technical questions; F. Meraldi mounted the corner reflectors; and J.J. Giganti and J. Mathews did some electronic design and construction.

A detailed account of the measurements is contained in two University of Maryland Ph.D. dissertations:

R. E. Williams, "A Direct Measurement of the Relativistic Effects of Gravitational Potential on the Rates of Atomic Clocks Flown in an Aircraft," (May, 1976).

R. A. Reisse, "The Effect of Gravitational Potential on Atomic Clocks as Observed with a Laser Pulse Time Transfer System," (May, 1976).

The work has also been described in talks at meetings and symposia, including: Precise Time and Time Interval Meeting, Goddard Space Flight Center, December, 1976; 30th Frequency Control Symposium, Atlantic City, June, 1976 (During Round Table Discussion on Remote Time Comparison); Second Symposium on Frequency Standards and Metrology, Copper Mountain, Colorado, July, 1976; Meeting of Commission 31 (Time) at the XVI General Assembly of the International Astronomical Union, Grenoble, France, August, 1976; Meeting on Experimental Gravitation, Sponsored by the Accademia Nazionale dei Lincei, Pavia, Italy, September, 1976; Symposium on Time and Frequency at the 19th General Assembly of the International Union of Radio Science, Helsinki, Finland, August, 1978.

37. The support of the U.S. Navy has come from many organizations in addition to the U.S. Naval Observatory, including its Directors Kai Strand and Gart Westerhout. These include:

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- Norris Keeler, Director of Navy Technology

The Office of Naval Research
- John Dardis, Project Monitor
- Doran Padgett, Project Monitor
- William Condell, Physics Branch Chief
- Fred Quelle, Boston Office

The Naval Air Development Center
- Kenneth Lobb, Technical Director
- Ronald Vaughn, Richard Sheklin } Navigation Laboratory

Patuxent Naval Air Test Center
- Robert Merritts, Project Engineer (He worked night and day along with the principal participants and contributed much to the success of the experiments).
- Pilots, Flight Engineers, Crew Chiefs, and Airmen for P3C-912.

38. The modifications were carried out at Maryland with R. Hyatt and J. Bourdet of Hewlett-Packard assisting L.S. Cutler and the Maryland group.

39. Frequent assistance in maintenance was provided by D. Kaufman and J. Soucy of the Goddard Space Flight Center.

40. We are indebted to Ernst Jechert and Hans Badura of Efratom for the loan of these standards.

41. The development of this experiment is described by C.O. Alley in Adventures in Experimental Physics, Alpha, 1972. Edited by B. Maglich. The major scientific result of the lunar laser ranging experiment is a test of the Principle of Equivalence for massive bodies (Earth and Moon both falling to the sun): J.G. Williams, R.H. Dicke, P.L. Bender, C.O. Alley, W.E. Carter, D.G. Currie, D.H. Eckhardt, J.E. Faller, W.M. Kaula, J.D. Mulholland, H.H. Plotkin, S.K. Poultney, P.J. Shelus, E.C. Silverberg, W.S. Sinclair, M.A. Slade, and D.T. Wilkinson, Physical Review Letters, 36, page 551 (1976).

42. We are indebted to Dr. Frank Hoge of the Wallops Center of NASA for the loan of this telescope.

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- Dr. Bernard Kulp, Chief Scientist

The Air Force Office of Scientific Research
- Major Richard Gullickson, Project Monitor
- Col. R. Detwiler, Physics Branch Chief

4950th Test Wing, Wright Patterson Air Force Base
- Lt. Steve Stratton, Test Director (He worked closely with the major participants, contributing much to the success of the measurements).
- Col. William Odgers, Commander
- Instrumentation Branch
- Modification Center
- Pilots, Navigators, Flight Engineer, Loadmasters, Crew Chiefs, and Airmen for C141 -- 779.

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45. B. Hoffmann, "Noon - Midnight Red Shift," Physical Review, 121, page 337 (1961).

46. R.U. Sexl, "Seasonal Differences Between Clock Rates," Physics Letters, 61B, page 65 (1976).

47. W.H. Cannon and O.G. Jensen, "Terrestrial Timekeeping and General Relativity: A New Discovery," Science, 188, page 317 (1975). The errors in this paper have been pointed out in many letters in a subsequent issue of Science: "Acceleration and Clocks," Science, 191, pages 489-491 (1976). The authors have retracted their claims.

48. R.U. Sexl, private communication, August, 1977.

49. R.F.C. Vessot and M.W. Levine, Gravitazione Sperimentale, Accademia Nazionale dei Lincei, Rome, page 371 (1977).

50. R.F.C. Vessot, presented at Second Marcel Grossmann Meeting on Recent Developments in General Relativity, Trieste, Italy, July, 1979.

51. L. Briatore and S. Leschiutta, "Evidence for the Earth Gravitational Shift by Direct Atomic-Time-Scale Comparison," Il Nuovo Cimento, 37B, No. 2 (1977).

52. S. Iijima and K. Fujiwara, "An Experiment for the Potential Blue Shift at the Norikura Corona Station," Annals of the Tokyo Astronomical Observatory, Second Series, Volume XVII, Number Z, (1978).

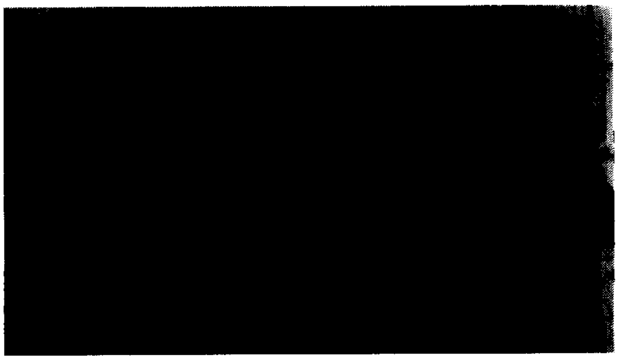
53. The NAVSTAR/Global Positioning System is fully described by seventeen articles in the GPS special issue of Navigation, Journal of the Institute of Navigation, Vol. 25, No. 2, Summer, 1978.

54. T. McCaskill, J. White, S. Stebbins, and J. Buisson, "NTS-2 Frequency Stability Results," Proceedings of the 32nd Frequency Control Symposium, 1978.

1. Albert Einstein in his study in Berlin.

2. Herblock drawing in the Washington Post after Einstein's death in 1955.

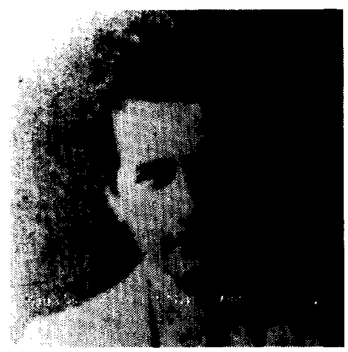
3. The earliest known picture of Einstein - age around 5 years.



4. Einstein in elementary school in Munich at the age of 10 (Second from right, first row).

5. Einstein at the age of 14.

6. The classroom of Dr. Jost Winteler in Aarau. Einstein at age 16 is at right.

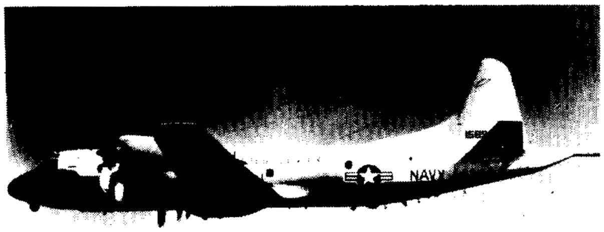


7. Einstein as a student at the ETH in Zurich.

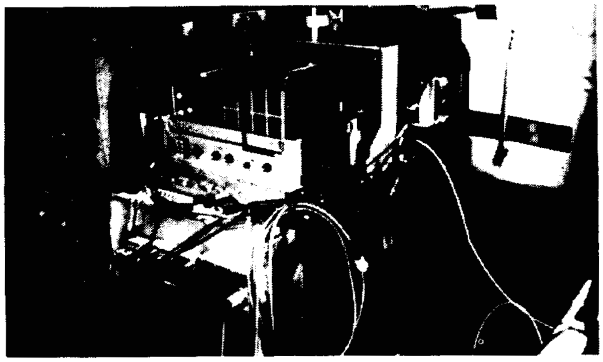
8. Einstein at his desk in the Swiss Patent Office in Bern at age 26.

9. Einstein in a playful mood at the California Institute of Technology in the early 1930's.

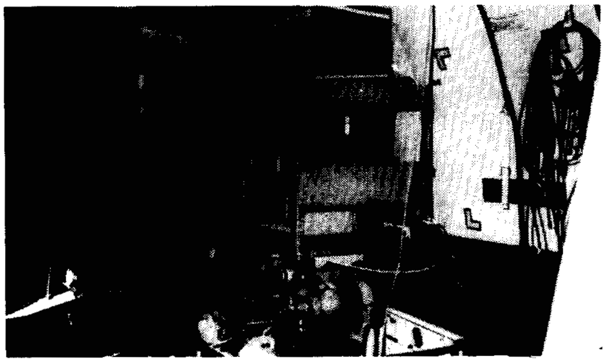
10. Einstein in his later years at the Institute for Advanced Study in Princeton.



11. Navy P3C Orion Aircraft 912 used in the experiments.

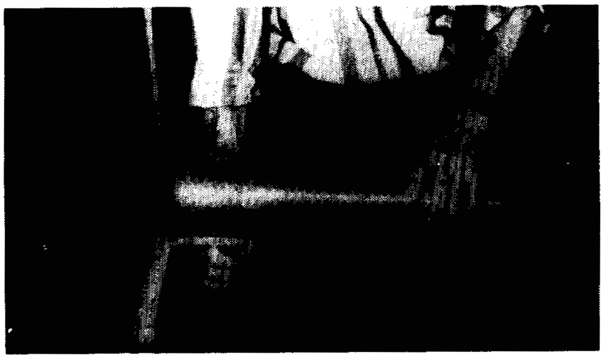


12. Clock Box installed on aircraft.

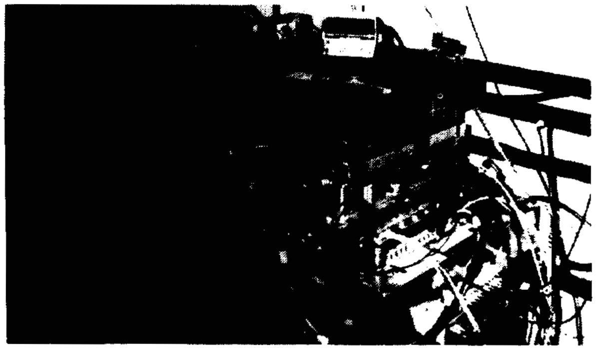


13. Electronic racks installed on aircraft.

14. Optical corner reflector on the aircraft of the type used for the Apollo Lunar Laser Ranging Retro-Reflectors.

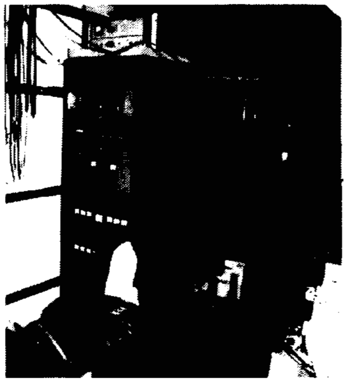


15. Housing of photo-multiplier for detection of laser light pulses on the aircraft.

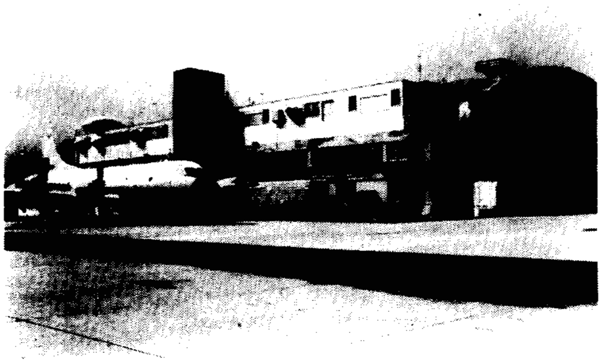


16. Clock Box in trailer on ground.

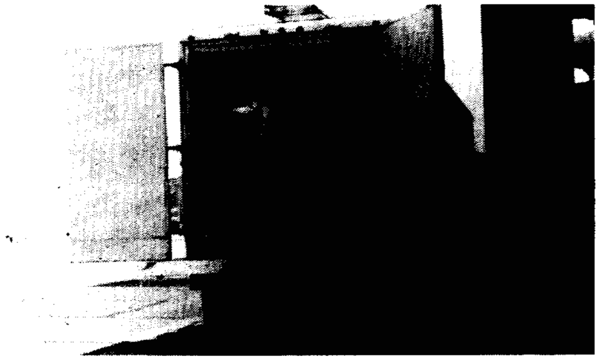
17. Ground computer system, event timer, and strip chart monitoring displays in trailer. (R.E. Williams, left and R.A. Reisse).



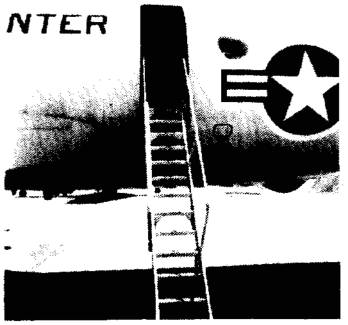
18. Two hydrogen masers made by Harry Peters on loan from the Goddard Space Flight Center.



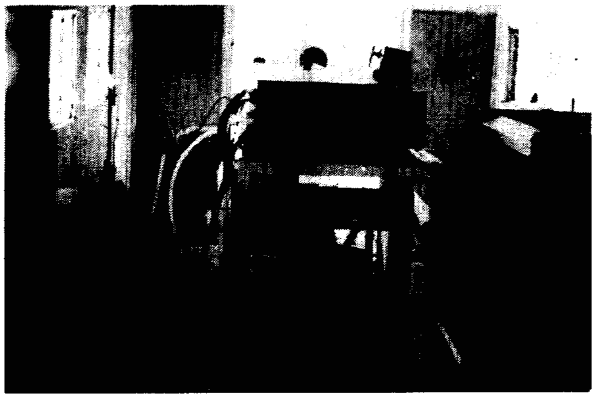
19. Aircraft on ground near trailer containing atomic clocks and van containing laser equipment.



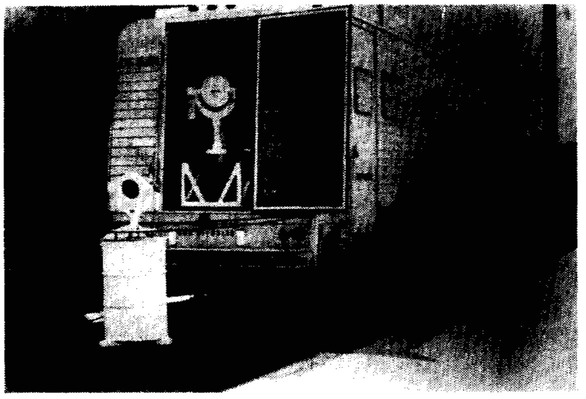
20. Clock Box being removed from trailer.



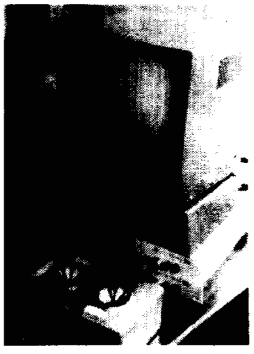
21. Narrow hatch of aircraft.



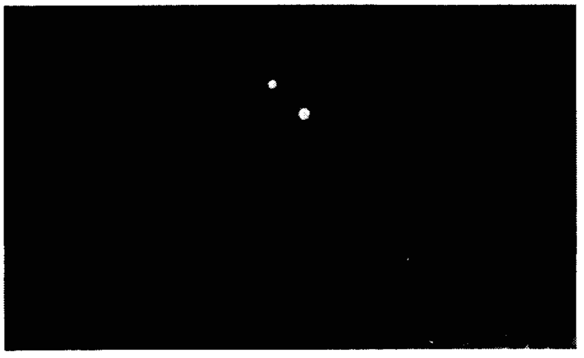
22. Laser Beam directing and receiving optics.



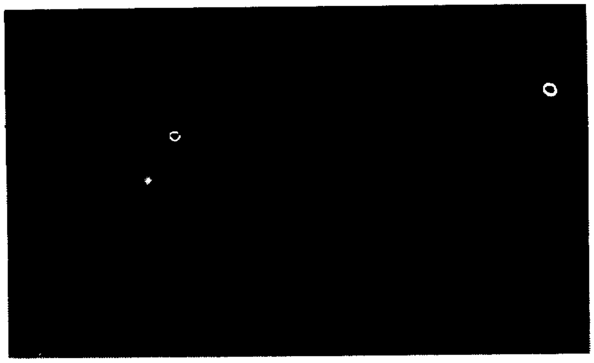
23. Laser equipment and receiving optics in van.



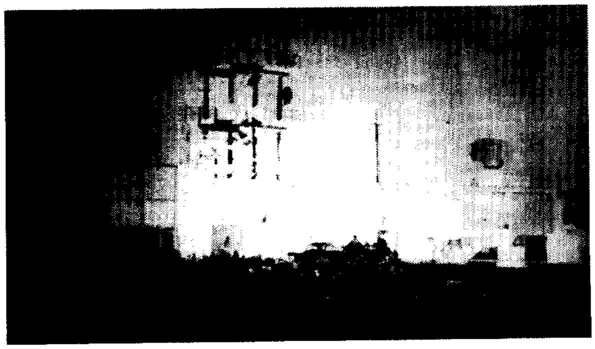
24. Closed circuit TV display with coarse and fine joysticks for aircraft tracking.



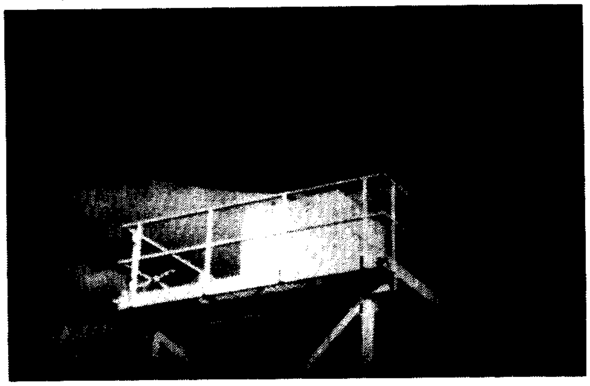
25. Landing lights of plane on TV display during tracking.



26. Laser transmitter and runways as seen from the airplane at twilight.

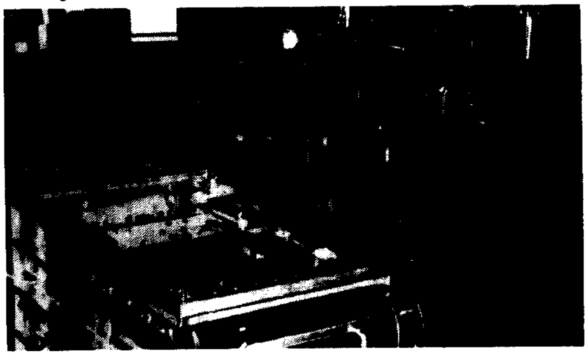


27. Chesapeake Test Range.

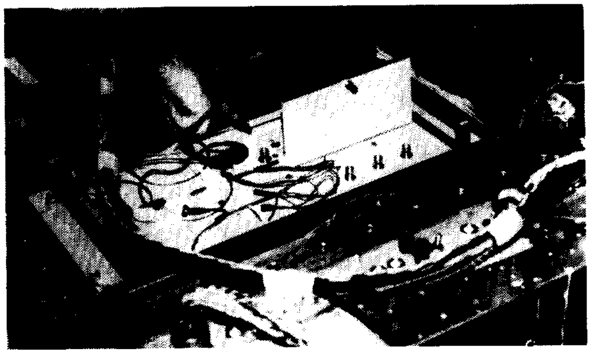


28. Ground tracking antenna at the Chesapeake Test Range.

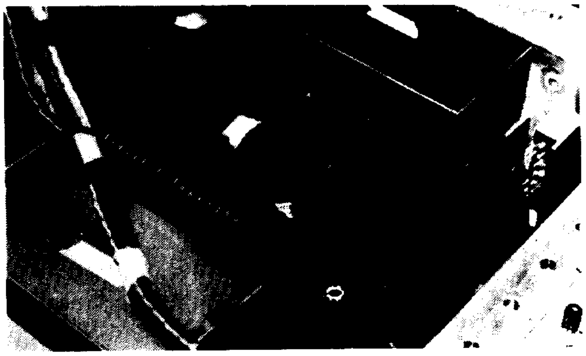
29. L.S. Cutler adjusting ensemble of modified Hewlett-Packard 5061A atomic clocks.



30. Interior of Clock Box.

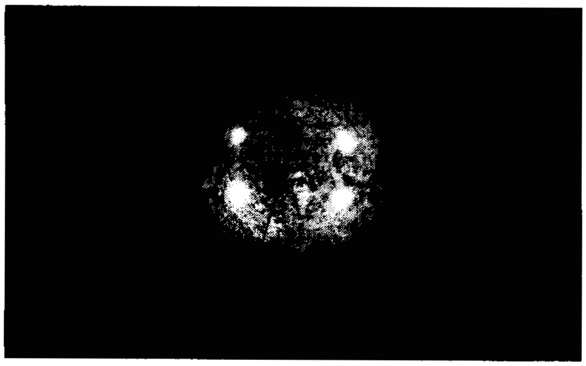


31. Clock inserted in its magnetic shield.



32. Air flow hoses for controlling temperatures and removing heat from clocks.

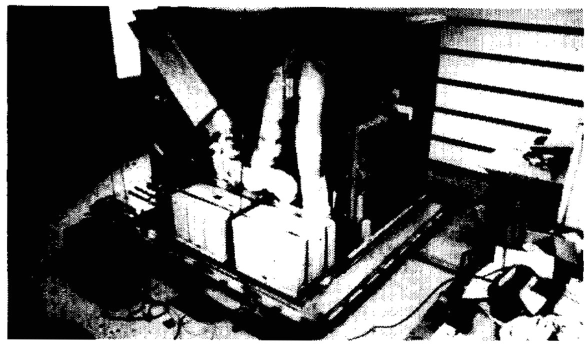
33. Lid carrying Efratom rubidium atomic clocks and voltage and pressure regulation equipment.



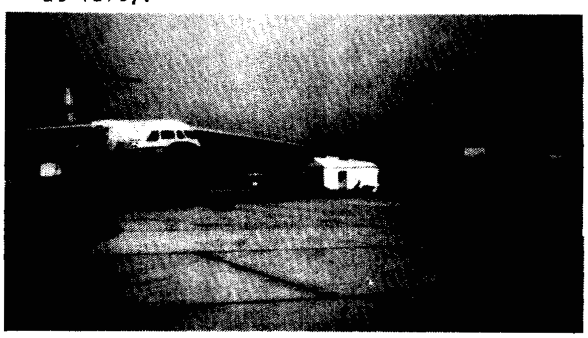
34. Polar photograph of globe showing distances of Washington and Thule from the Earth's spin axis.

35. Airborne equipment being reconfigured for C141 cargo pallet (Front View).

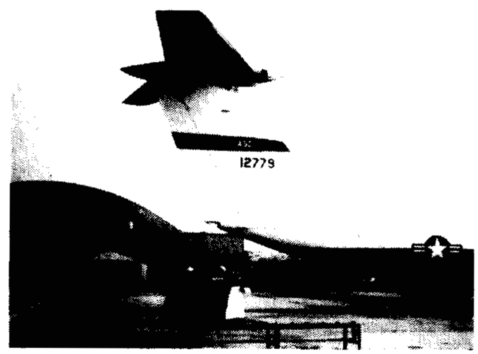
36. Airborne equipment being reconfigured for C141 cargo pallet (Back View) (Technician Lyndon Small at left).



37. Airborne equipment on cargo pallet housed in pre-fabricated garage of Andrews Air Force Base.

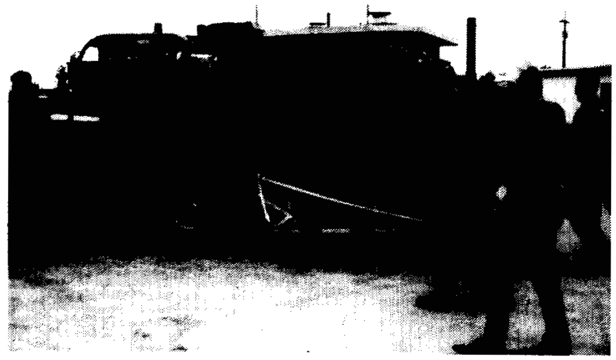


38. Air Force C141 Starlifter Aircraft 779 at Andrews Air Force Base next to garage, trailer, and van.



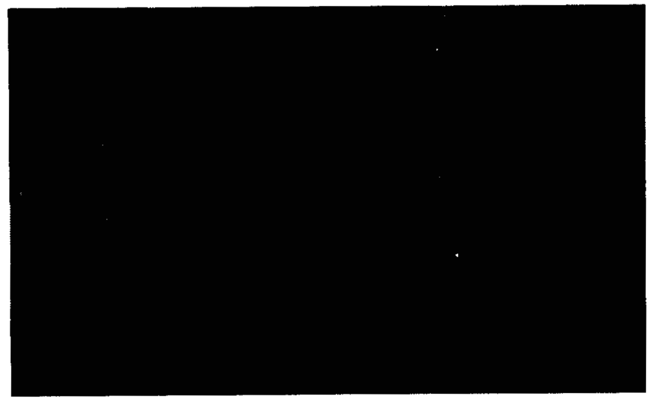
39. Tail of C141 with petal doors through which equipment was loaded.

40. Preparing to transfer equipment from garage.



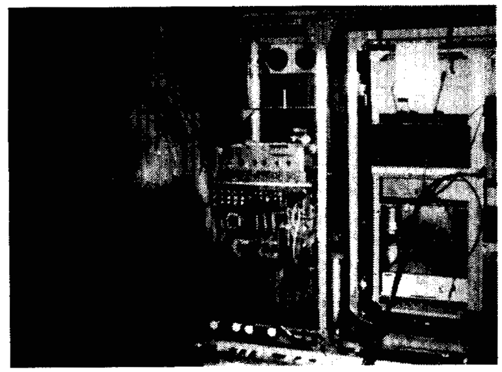
41. Front lift carrying pallet to aircraft.

42. Petal door open to receive pallet.



43. Matching of pallet to aircraft roller tracks.

44. Rolling pallet to forward part of aircraft.



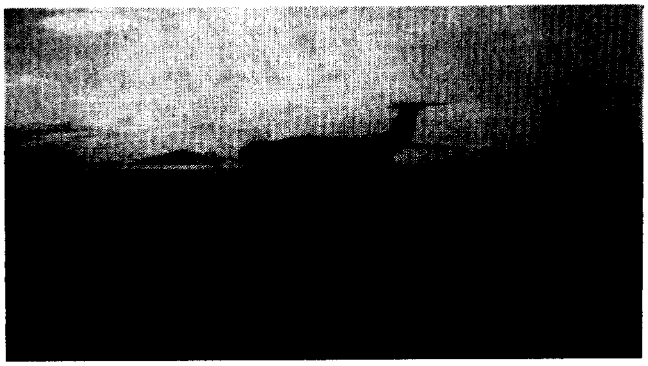
45. Len Cutler beside installed pallet on aircraft.

46. Clock Box Assembly during flight. (Seen from rear of plane).

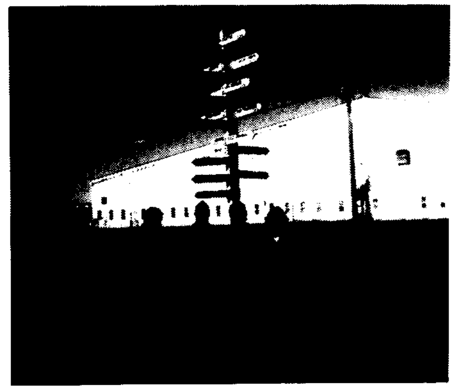
47. Clock Box Assembly during flight. (Seen from front of plane).

48. Recording and readout equipment for inertial navigation units and altimeters.

49. Preparing to place aircraft in hangar at the Thule Air Force Base.



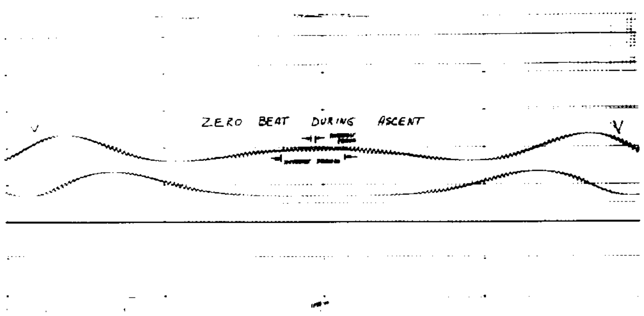
50. A C141 at Thule just before midnight on June 25.



51. Midnight shadows cast by sun at Thule on June 25.

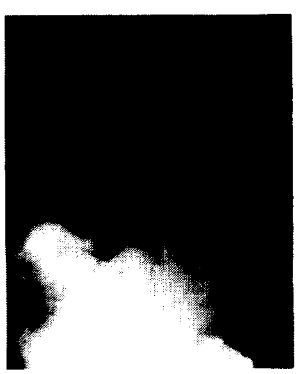
52. Globe tilted at $23^{\circ}.5$ showing relation between Washington and Christ Church at time of summer solstice (Sun to right).

53. C141 779 on ground at Christ Church surrounded by support equipment.

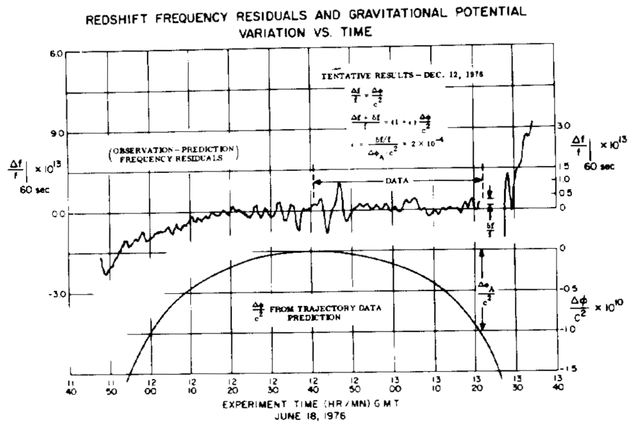


54. Zero beat during ascent of hydrogen maser on rocket probe.

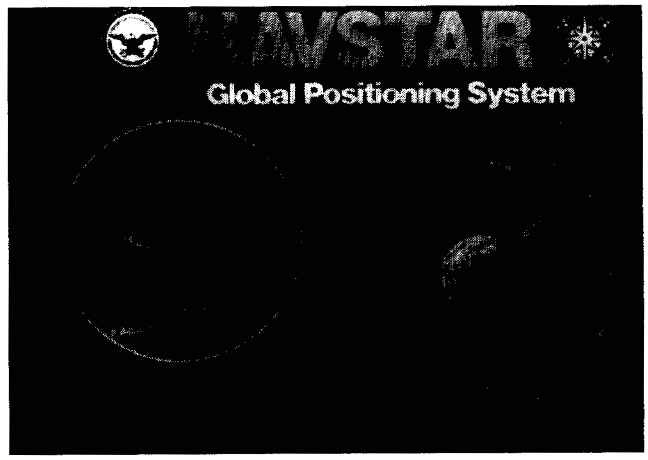
55. Environmentally packaged hydrogen maser to be carried by Scout rocket being held by R. F. C. Vessot (right) and M. Levine.



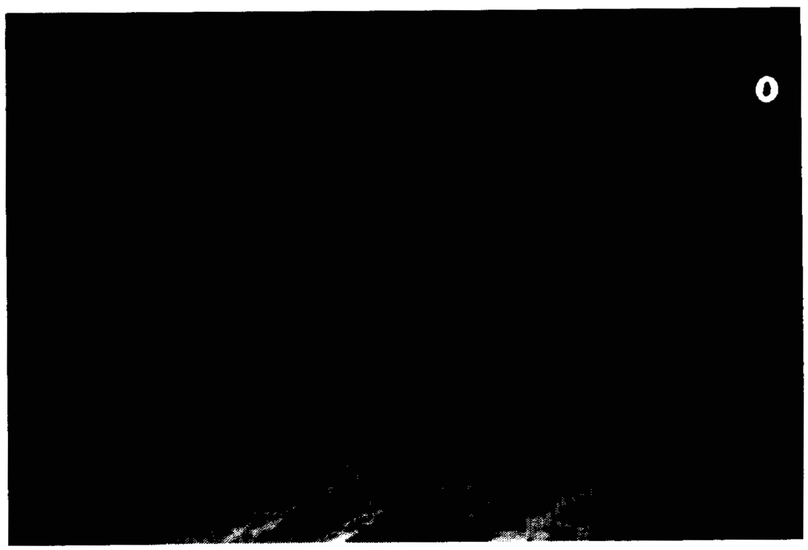
56. Launch of Scout rocket carrying hydrogen maser.



57. Residuals during an early stage of the analysis of the hydrogen maser rocket probe data.



58. Artist's drawing of NAVSTAR/Global Positioning System.



59. Major segments of the NAVSTAR system.