Why Did Newton See Indigo in the Spectrum?

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The arrangement of colors in Newton's color circle suggests that it was derived from paint mixtures, not light mixtures. If this is true, we are led to conclude that what Newton called indigo represents violet in modern terminology, and what he called violet represents purple. The author hypothesizes that Newton saw seven reasonably distinct colors in the artist's paint mixture color circle (red, orange, yellow, green, blue, violet, and purple) and therefore assumed he could also see seven distinct colors in his crude spectral projections.

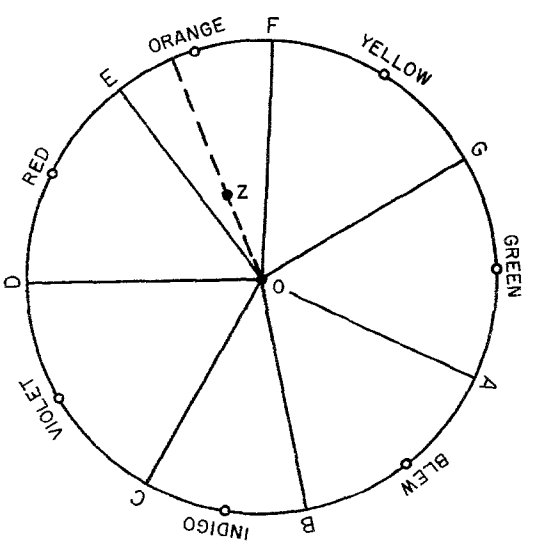

FIG. 1. Newton's color circle.

1. NEWTON'S COLOR CIRCLE

Newton claimed that there are seven colors in the spectrum: red, orange, yellow, green, blue, indigo, and violet; however, anyone who honestly looks at the spectrum cannot see more than six distinctly different colors. There is no indigo. Of course, the colors of the spectrum change in a continuous way from red to violet, and if one insists that there must be a color between blue and violet, an indigo can be forced into that region. Many people in describing the prismatic colors continue to do this, assuming that if the great Sir Isaac Newton said there is an indigo, it certainly must be there.

Recently I was examining Newton's color circle carefully and discovered that it is a truly remarkable diagram. I think it holds the key to the mystery of indigo. Let me present my hypothesis.

Figure 1 shows Newton's color circle, which is Fig. 11 in Book One, Pt. II of Newton's *Opticks*,[1] first published in 1704. It has generally been assumed that Newton derived his color circle on the basis of additive mixtures of colored lights that he formed by means of prisms. If this were true, however, he would have placed blue opposite to yellow, because in an additive mixture of colored lights blue and yellow are complementaries: Their mixture forms grey. In Newton's circle, however, blue is opposite to orange. That is the arrangement that occurs in a so-called "subtractive" color mixture that results from mixing paints. I suspect, therefore, that Newton did not invent his color circle at all; he probably borrowed it from an artist and applied it to his problem.

2. DIFFERENCE BETWEEN COMPLEMENTARY AND CONTRASTING COLORS

Let me digress for a moment and explain some important anomalies about color circles. Artists today generally use the color circle shown in Fig. 2, which was first published by Moses Harris in 1766 as a hand-painted color reproduction in

his book, *The Natural System of Colours.*[2] This same circle was used by Goethe[3] in his *Theory of Colours,* published in 1810, in which Goethe strongly attacked Newton's color doctrine. Goethe's theoretical explanation of the prismatic colors is in contradiction with scientific facts, but his interpretation of visual contrast phenomena has greatly influenced our modern concepts of color harmony.

There are two reasons for the artist using the color circle of Fig. 2: (1) Opposite colors are approximate after images of one another, and (2) paints mix approximately in accordance with this color circle. After-image color pairs provide maximum simultaneous contrast as well as maximum successive (after-image) contrast. It is these contrast processes, being performed in the retina of the eye, that are essential features underlying color harmony. A recent book by Itten[4] has provided an excellent discussion from the artist's point of view of the relationship between the color circle and color harmony. It is extensively illustrated with magnificent color reproductions. Itten employs the Harris color circle of Fig. 2, but uses the word "violet" where Harris and Goethe used "purple."

When colors are mixed by combining lights in an additive color mixture, the color circle takes on the form of Fig. 3 if the colors are highly chromatic. Opposite colors of this color circle (which I call complementary colors) produce grey or white when combined as lights, whereas

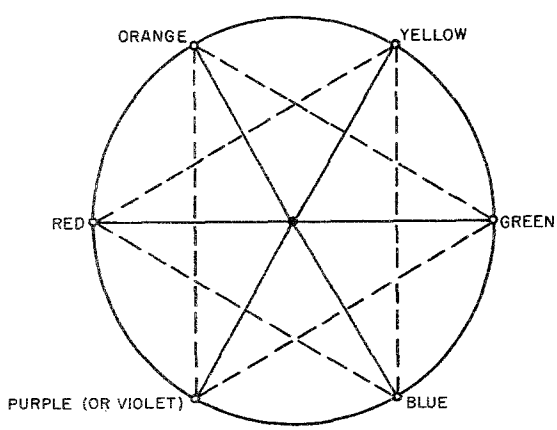

FIG. 2. The Harris color circle used by Goethe and artists of today.

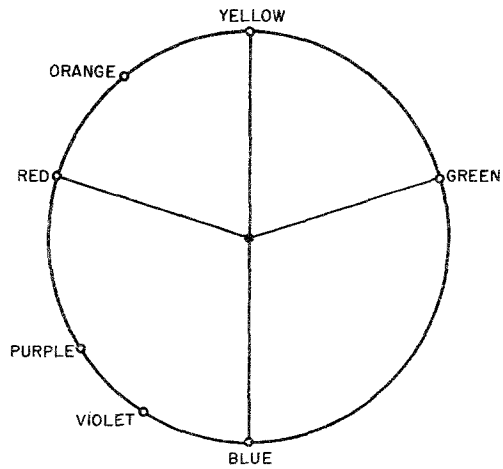

FIG. 3. Color circle based on additive light mixtures.

opposite colors of the artist's color circle of Fig. 2 (which I call contrasting colors) approximately produce grey in a paint mixture.

Wilson and Brocklebank[5] have performed a careful study of the difference between the complementary colors of an additive color mixture and the contrasting colors observed in after images. They explained the anomalous differences between these color circles by showing that when some of the chromatic colors are desaturated with white or grey in an additive light mixture, the hue changes strongly, particularly in the blue and yellow regions. The blue light shifts toward violet, and the yellow light shifts toward orange. Consequently, when complementary blue and yellow lights are mixed, the two hues shift such that, near the point where the mixture turns grey, the hues are orange–yellow and blue–violet, which are contrasting colors.

For red and green lights there is practically no shift in hue as the color is desaturated, so that contrasting and complementary colors are practically the same in these regions. As shown in Fig. 3, red is not the exact complement of green (nor is it the exact after image of green). The complementary and contrasting color (after image) of red is bluish–green, and the complementary and contrasting color of green is magenta (purplish–red). Note that the Harris color circle of Fig. 2 is deficient in the red and green regions because it places red opposite to green, even though they are not exactly contrasting colors.

3. HOW DID NEWTON OBTAIN HIS COLOR CIRCLE?

Now, let us look back at Newton's color circle in Fig. 1. This is an excellent circle for showing contrasting colors if we make the assumption that Newton's "indigo" is what we generally call violet, and Newton's "violet" is what we generally call purple. In fact, Newton's color circle gives a significantly better indication of after-image pairs (and probably also paint mixtures) than does the Harris color circle used by Goethe and most artists of today.

My hypothesis is, therefore, as follows: Newton obtained his color circle from a painter. On the painter's color circle Newton saw seven distinct hues: red, orange, yellow, green, blue, violet, and purple. He assumed that all these colors must be in the spectrum and so deceived himself into believing that he could also see seven distinct spectral colors.

Let me emphasize that the choice of seven distinct hues in the full color circle is not at all arbitrary. The reader can verify this by examining a good color atlas, such as the *Munsell Book of Color*, which should be available at any art school. The eye perceives four unique hues, red, yellow, green, and blue, the location of which can be measured quite accurately by psychological judgements. Between green and its unique neighbors there are no distinct hues, merely blue–greens and yellow–greens. However, between red and yellow we perceive orange, which has a distinct character of its own. Between blue and red the character of the color changes drastically, so that one can clearly see two distinct intermediate hues, violet and purple. The actual positions of violet and purple cannot be measured precisely (as can the unique hues). Nevertheless, to provide a color circle with reasonably equivalent perceptual spacing, there should be two intermediate hues between red and blue (i.e., violet and purple), one intermediate hue between red and yellow (i.e., orange), and no intermediate hues between green and blue and between green and yellow.

The reader can verify for himself the points I have made concerning contrasting and complementary colors by means of simple experiments. For producing additive mixtures, one needs a wheel that rotates at a rate of at least 20 revolutions per second. Circular colored papers are placed on the face of the wheel, each cut along a radial line so that the papers can be interlaced. This allows one to change the sectors covered by the different colors and thereby to vary the mixture. A clear plastic cover should be placed over the papers to protect them during the rotation. To determine contrasting colors, one should stare at a colored paper for about 30 sec, and then see the after image by looking at a uniform medium grey background. For both experiments, brightly colored papers should be used. The best illumination is indirect daylight, particularly that from the north sky.

Mixing with a color wheel is much simpler than combining lights by projection from prisms as Newton did. On the other hand, the results are not quite equivalent because the colored papers on the wheel are not as chromatic as the spectral colors from a prism. Nevertheless, with brightly colored papers, additive light mixtures can be obtained that are reasonably consistent with those achieved using spectral lights.

It should be recognized that a large section of the hues of the color circle cannot be seen in the spectrum. We perceive the longest visible wavelength as a slightly orangish–red and the shortest wavelength as a violet. The nonspectral hues are formed by mixing lights from both the blue and red ends of the spectrum. Such colors as pure red, magenta, and purple are nonspectral colors.

When Newton related his spectrum to the artist's color circle, however, he folded the spectrum completely around the circle, leaving no gap for the nonspectral hues. He shows this clearly in the following discussion of Prep. VI Prob. II in *Opticks*, where he describes his color circle shown above in Fig. 1:

> Let the first Part *DE* represent a red Colour, the second *EF* orange, the third *FG* yellow, the fourth *GA* green, the fifth *AB* blue, the sixth *BC* indigo, and the seventh *CD* violet. And conceive that these are all the Colours of uncompounded light gradually passing into one another, as they do when made by Prisms; the Circumference *DEFGABCD*, representing the whole Series of Colours from one end of the Sun's colour'd Image to the other.

Notice that Newton describes the segments of his circle as being "the colours of uncompounded light" (i.e., the colors of rays of a single wavelength). He also describes the full "Circumference *DEFGABCD*" of the circle as representing the complete light spectrum. Thus Newton left no space in his color circle for nonspectral hues, and so it is reasonable to assume that he felt compelled to include in the spectrum all of the seven distinct hues that he saw in the artist's color circle.

Newton does discuss a "nonspectral" color, purple, but he does so in an anomalous way. In Prop. IV, Theor. III he states:

> Lastly, if red and violet be mingled, there will be generated according to their various Proportions various Purples, such as are not like in appearance to the Colours of any homogeneal Light, and of these Purples mix'd with yellow and blue may be made other new Colours.

In his description of the color circle in Prop. VI Prop. II, he states:

> But it is to be noted, that if the point *Z* fall in or near the line *OD*, the main ingredients being the red and violet, the colour compounded shall not be any of the prismatic Colours, but a purple, inclining to red or violet, accordingly as the point *Z* lieth on the side of the line *DO* towards *E* or towards *C*, and in general the compounded violet is more bright and more fiery than the uncompounded.

(In this discussion, the point *Z* is the "common Century of Gravity" of the rays of the color mixture.) In the first passage, Newton speaks of various purples between red and violet and also "other new Colours" when these purples are mixed with yellow and blue. Nevertheless, he does not designate any separate area in the color circle for purple or these "other new Colours." In the second passage he describes purple as being a "compounded violet" that is "more bright and more fiery than the uncompounded." This suggests that Newton regarded purple and the "other new Colours" as being brighter variations of the uncompounded colors.

4. THE CONTROVERSY CONCERNING NEWTON'S THEORY OF COLORS

To place Newton's thinking in perspective, I would like to quote E. T. Whittaker[1] in his "Introduction" to the 1931 republication of Newton's *Opticks*:

> It was extraordinarily difficult for even the most brilliant of Newton's contemporaries to accept his doctrine that light existed in an infinite number of different independent colours, incapable of being changed into one another, and characterized by a definite refrangibility. He seemed to be refuted by experiments in which a mixture of paints of two colours produced paint of a third colour, and by other experiments belonging to the subjective physiological theory of colour-vision, in which colours really are compounded out of primaries. Even Huygens in 1673 said that "an hypothesis which should explain the colours yellow and blue, would be sufficient for all the rest."

Thus, we see that Newton's fundamental problem was to relate his prismatic colors to the color mixture effects produced by the artist when mixing paints. It is only natural, therefore, that Newton would use an artist's color circle to explain his prismatic colors.

Newton explained color mixture effects by assuming that the combination of the different rays of the spectrum produces the effect of a white plus a pure spectral light, which is defined by the "center of gravity" of the rays of his color circle. The pure spectral light is sensed by the eye to give the hue, and the closeness of the center of gravity to the center of the circle is sensed to give the amount of whiteness. Obviously, if Newton left a gap in his color circle for the nonspectral hues, such as purple and magenta, there would be no spectral ray that he could associate with that color; therefore, he apparently left no gap, and deceived himself into believing that the colors of the spectrum would fold completely around the artist's color circle.

We should realize that Newton was not interested in colors per se. He was concerned with a much more basic physical phenomenon, the nature of light. In those days, however, the eye

was the only means of measuring light, and color mixture effects represented the standard of physical reality against which Newton's concept of the light spectrum was being compared—his concept that the spectrum is continuous and that rays from different parts of the spectrum are independent and cannot be changed into one another. It is obvious to us that Newton's prism experiments proved his point very strongly and were much more fundamental than color mixture effects. To the scientists of Newton's day, however, color changes produced by mixing paints represented the physical reality to which they were accustomed, and they dismissed Newton's results as being mere anomalies because they were in conflict with their preconceived notions of physical reality.

Newton first presented his concept of the spectrum in a paper to the Royal Society of London on 6 Feb. 1672. As Cohen[6] has explained, Newton naively assumed he would be applauded for his results but instead received a series of major criticisms by Hooke, Huygens, Moray, Pardies, Linus, Gascoines, and Lucas. Newton responded to these comments, one by one, and soon found himself a "slave to philosophy," and planned "resolutely [to] bid adieu to it eternally, excepting what I do for my private satisfaction or leave to come out after me; for I see a man must either resolve to put out nothing new, or to become a slave to defend it." As a result of this opposition, Newton did not publish his book *Opticks* until 1704 (32 years later!), even though part of the book was completed in 1675 and most of the rest by 1687.

With all of these psychological pressures on Newton, it is easy to see how he could be led to believe the complete hues of the artist's color circle were in his spectrum. That assumption would make his theory much simpler, and he was weary from the emotional nit-picking attacks he had received from the scientists of his day.

5. NEWTON'S MEANING OF THE COLOR NAMES

To explore further what Newton meant by his color names, let us examine the earlier work on this subject published in the *Philosophical Transactions of the Royal Society of London*[7] between 1672 and 1676, which includes his original paper and the subsequent debate on his theory with members of the Royal Society. In this material, Newton used the words "violet" and "purple" synonymously. For example, his reply to Linus published in March 1676 describes the spectrum projected by the Sun's image through a prism with the words:

> the Colours Red, Yellow, Green, Blue, Purple, succeeding in Order....

In his later reply to Gascoines, also published in March 1676, he describes the same image by:

> they will see the Image...with the Red at one End, the Violet at the other, and the bluish Green in the Middle.

In these two passages, written a few months apart, Newton interchanged the words "violet" and "purple" for exactly the same meaning. In his original paper, presented in 1672, he stated:

> There are therefore two sorts of Colours, the one Original and Simple, and the other compounded of these. The original or primary Colours are, Red, Yellow, Green, Blue, and a Violet–Purple, together with Orange, Indigo, and an indefinite Variety of intermediate Gradiations.

Here Newton combines violet and purple together as a single color, violet–purple.

In this early material from 1672–1676, there is no reference to the notion that purple is a nonspectral color, nor does Newton make any reference to his color circle. That material was evidently written about 1687.

The historical definitions of the words violet, purple, and indigo in the *Oxford English Dictionary*[8] yields the following information. Violet was a rather vague term, defined as having the color of violets (which varies widely). Purple evolved in ancient times as the name for a dye obtained from the Purpura mollusk, and was actually crimson (purplish–red) in color. In the middle ages, "purple" was applied vaguely to many shades of red. The color became the dress for emperors and had the characteristics of "bright-hued" and "brilliant." Indigo (or indico) was the color of a "blue" Indian dye from a plant of the same name and was a fairly recent term in

Newton's day. A quote in 1826 says: "Good indigo has a deep blue color inclining to purple."

Thus, we conclude that in Newton's day purple was probably at least as near to red as our modern definition for purple. The fact that Newton used the word purple synonymously with violet in his early writings suggests that he believed, at least at that time, that the non-spectral color region that we now designate as purple fell within the spectrum. The word violet probably had a vague meaning at that time; it is much more precise now because our definition of violet is based on Newton's use of that word in *Opticks* as the color of the shortest visible wavelengths. The meaning of indigo to Newton can probably be clarified by examining the range of hues produced by indigo dye as it was presumably formulated in Newton's time.

6. CRUDITY OF NEWTON'S COLOR MIXTURE EXPERIMENTS

It is important to recognize that Newton's color mixture experiments with prismatic lights were very crude, as can be seen from the following Prop. VI, Pt. II of *Opticks*[1]:

> Also if only two of the primary Colours which in the circle are opposite to one another be mixed in an equal proportion, the point Z shall fall upon the center O, and yet the Colour compounded of these shall not be perfectly white, but some faint anonymous Colour. For I could never yet by mixing only two primary Colours produce a perfect white. Whether it may be compounded of a mixture of three taken at equal distances in the circumference I do not know, but of four or five I do not much question but it may. But these are Curiosities of little or no moment to understanding the Phenomena of Nature. For in all whites produced by Nature, there uses to be a mixture of all sorts of Rays, and by consequence a composition of all Colours.

Obviously, if Newton had performed this experiment carefully, he would have discovered that blue and yellow spectral lights mix to produce white, not green.

Two paragraphs later, Newton described the experiments on which his color circle was presumedly based as follows:

> This Rule I conceived accurate enough for practice, though not mathematically accurate; and the truth of it may be sufficiently proved to Sense, by stopping any of the Colours at the Lens in the tenth Experiment of this Book. For the rest of the Colours which are not stopp'd, but pass on to the Focus of the Lens, will there compound either accurately or very nearly such a Colour, as by this Rule ought to result from their Mixture.

In the particular experiment referenced here (Exper. 10, Book I, Pt. II), the spectral resolution was very poor. The Sun's image projected through a small hole was allowed to fall directly on the prism, and the resultant spectrum projected by the prism was only five times as long as it was wide because of the divergence of rays from different parts of the Sun's disk. As a result the wavelengths of the spectrum were smeared over about $\frac{1}{4}$ of the spectral band.

The visible wavelength band is about 425 nanometers (nm) to 675 nm, which gives a wavelength spread of 250 nm. Hence in Newton's experiment the wavelengths were smeared over a band of (250 nm/4), or about 60 nm. For comparison the following are the approximate wavelength ranges of the spectral colors: 425–450 nm are violet, 450–480 nm are blue, 480–560 are green, 560–590 are yellow, 590–620 are orange, and 620–675 are red. (See Ref. 3, p. viii.) We see that the 60-nm wavelength smear in Newton's experiment was as great as the total yellow-plus-orange band or the total blue-plus-violet band. Thus, the spectral resolution of Newton's color-mixing experiment was very poor.

In other experiments, Newton achieved much better spectral resolution by placing a lens in front of the prism, so that the image of the Sun was brought to focus on the spectrum cast by the prism. (See, for example, Exper. 11 of Book I.) However, he apparently did not use this procedure in his color mixing experiments. It should also be noted that all of his earlier experiments reported in the *Philosophical Transactions of the Royal Society* were performed with the same low spectral resolution as his color-mixing experiments.

7. VARIATIONS OF SPECTRAL COLORS WITH VIEWING CONDITIONS

The preceding discussion has described various wavelengths as having certain colors. This is a simplification, however, because the color evoked by a particular wavelength varies with the state of adaptation of the eye. When the eye is adapted to daylight and is looking at a narrow wavelength band, the spectrum extends from an orangish–red to violet. When one perceives the spectrum as a whole, however, simultaneous contrast effects make the color spread of the spectrum appear greater, and in addition, it is difficult to determine the full extent of the spectral colors because the blue and red ends are quite faint.

One can also see spectral colors when looking through a prism at a scene and under this condition simultaneous contrast effects are even stronger. Color fringes occur at dark–light edges of the scene and are produced maximally when the edges are parallel to the edges of the prism. If the thick part of the prism is held on the white side of a black–white edge, the color fringe varies from blue to purplish–violet, whereas, if the prism is reversed, the colors vary from yellow to magenta. When one looks at a complicated scene, the various color fringes mix so that one can see the full color circle in the fringes. Thus, considering all of these complicated color perception effects, it is easy to understand how Newton could have convinced himself that he could see the full perimeter of the artist's color circle in the spectrum.

8. RELATION TO EARLIER STUDIES OF NEWTON'S WORK

Recent critical evaluations of Newton's spectral experiments have been presented by Lohne[9] and Holtzmark.[10] In particular, Lohne shows that Marci performed very similar experiments in Prague, which he published in 1648, and came to essentially the same conclusions about the spectrum. Marci's work was not circulated, however, and Newton and his contemporaries in England were unaware of it.

On the other hand, neither Lohne or Holtzmark touch on the question being raised in this paper of the anomaly of Newton's color circle, and the matter of the meaning of his color names. As far as the writer has been able to determine, no one has pursued this issue previously, although it is well recognized that Newton's color circle gives a very poor description of additive color mixtures.

It is to be noted that the explanation that has been given here as to why Newton saw indigo in the spectrum is presented merely as an hypothesis, which appears to reconcile a number of seemingly contradictory points. It is hoped that others will examine the question carefully and express their own conclusions.

9. RELATION BETWEEN PAINT-MIXTURE AND COLOR-CONTRAST EFFECTS

As a final point, I would like to present an hypothesis to explain why paint-mixture and color-contrast effects behave according to the same color circle, a circle that is quite different from that which results from light mixtures.

Very complicated spectral changes occur when mixing paints if the paint particles strongly scatter the light. However, there is a large class of paints that mix in a mathematically simple way, in accordance with Beer's and Bouguer's laws. These paints consist of a suspension of very fine particles that absorb light but cause little scattering. The mixing of such paints produces the same effect as placing the individual paints on top of one another in separate layers, and so is equivalent to the superposition of separate colored filters.

The color change produced by superimposing colored filters is popularly called a "subtractive" color mixture, because adding a filter reduces the transmitted light. A more appropriate term, however, is a *multiplicative* color mixture since the transmissivity at any wavelength of the combined filter sandwich is equal to the product of the transmissivities of the individual filters at that wavelength. (Transmissivity is the ratio of transmitted to incident light.) Thus, when optical filters are superimposed, or when low-scatter paints are mixed, the resultant transmissivity spectrum is the product of the transmissivity spectra of the individual filters or paints; whereas when colored lights are mixed, the resultant light spectrum is the sum of the spectra of the individual lights.

I postulate that the visual adaptation processes

being performed within the retina of the eye, which are responsible for simultaneous and successive contrast effects, operate in terms of neural signals that are approximately proportional to the logarithm of the light stimulus. (This assumption is in agreement with the Weber–Fechner law.) A multiplication of light spectra would then be equivalent to an addition of the neural signals employed in visual adaptation, and hence a multiplicative color mixture would result in a summation of color components in the visual adaptation stage. Therefore, according to this postulate, the simultaneous and successive color-contrast effects caused by visual adaptation should follow the same color circle as a multiplicative color mixture, which results when low-scatter paints are mixed or when color filters are superimposed.

[1] I. Newton, *Opticks* (republished by Dover, New York, 1952). (This is a reprinting of a 1931 republication by G. Bell and Sons, Ltd., of the fourth edition of Isaac Newton's *Opticks*, originally published in 1730, and contains introductory material by Sir Edmund Whittaker, Albert Einstein, and I. Bernard Cohen.)

[2] F. Birren, *Color—A Survey in Words and Pictures* (University Books, New Hyde Park, N. Y., 1966).

[3] J. W. von Goethe, *Theory of Colours* (MIT, Cambridge, Mass. and London, 1970). (This is a republication of the 1840 English translation, with extensive notes, by C. L. Eastgate, of Pt. I of Goethe's *Farbenlehre*, originally published in Germany in 1810, and it includes an introduction by Deane B. Judd.)

[4] J. Itten, *The Art of Color* (Reinhold, New York, 1961). (This was originally published in Germany in 1961 under the title *Kunst der Farbe*, Otto Maier Vorlag Ravensburg, publishers, and was translated into English by Ernst van Haagen.)

[5] M. H. Wilson and R. W. Brocklebank, J. Opt. Soc. Amer. **45**, 293 (1955).

[6] I. B. Cohen, Arch. Intern. d'Histoire des Sciences **11**, (Oct.–Dec. 1958).

[7] *The Philosophical Transactions and Collections to the End of the Year MDCC—Abridged and Disposed under General Heads—Vol. I* (Royal Society of London, 5 May 1703, Fifth Edition, MDCCXLIX).

[8] *The Oxford English Dictionary* (Clarendon, Oxford, Eng., 1961).

[9] J. A. Lohne, Notes Rec. Roy. Soc. London **23**, 169 (1968).

[10] T. Holtzmark, Amer. J. Phys. **38**, 1229 (1970).

Necessary and Sufficient Conditions for a Canonical Transformation

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Necessary and sufficient conditions are given for a transformation to be canonical. It is shown that if the transformation is to be canonical for all Hamiltonians, the condition is equivalent to the usual fundamental Poisson bracket conditions.

INTRODUCTION

In most advanced texts on mechanics it is shown that if the fundamental Poisson bracket conditions

$$[Q_i, P_j]=\delta_{ij},$$
$$[P_i, P_j]=0, \tag{1}$$

and

$$[Q_i, Q_j]=0$$

are satisfied, that the transformation

$$Q_i=Q_i(q_1, \cdots, q_n, p_1, \cdots, p_n, t),$$
$$P_i=P_i(q_1, \cdots, q_n, p_1, \cdots, p_n, t) \tag{2}$$