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2016 Eur. J. Phys. 37 055303

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Wave and photon descriptions of light: historical highlights, epistemological aspects and teaching practices

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Received 16 May 2016, revised 24 June 2016

Accepted for publication 1 July 2016

Published 27 July 2016



CrossMark

Abstract

The wave and the photon descriptions of light are discussed in connection with interference and the Doppler effect. Both phenomena can be described in terms of waves or in terms of photons. However, the two descriptions are not equivalent. While the photon description deals with a single photon, the wave one deals with the behaviour of many (enough) photons. The wave description's predictions are in agreement with experiment only when the high number of photons used (one at a time or all together) overshadows their spatially localised interaction with the detectors. These arguments are dealt with on the basis of historical breakthroughs in the light epistemological considerations and teaching implementations.

Keywords: interference, Doppler effect, special relativity

(Some figures may appear in colour only in the online journal)

1. Introduction

The 19th century has witnessed the triumph of the wave description of light over the corpuscular one. The identification of light with electromagnetic waves, due to Maxwell, constituted a kind of seal of this process. However, in 1905, Einstein put forward the 'heuristic' hypothesis that light might be constituted by '*energy quanta that are localized in points in space, move without dividing, and can be absorbed or generated only as a whole*' [1, p 87]. Einstein stressed that 'optical observations apply to time averages and not to momentary

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values, and it is conceivable that despite the complete confirmation of the theories of diffraction, reflection, refraction, dispersion, etc, by experiment, the theory of light, which operates with continuous spatial functions, may lead to contradictions with experience when it is applied to the phenomena of production and transformation of light' [1, p 87].

The fundamental equation

$$E = h\nu \quad (1)$$

relating the energy of the light quantum to the frequency of the corresponding electromagnetic radiation does not appear in Einstein's paper because Einstein does not use Planck's constant h . As a matter of fact, Einstein arrives at an equation equivalent to (1) by comparing the entropy dependence on the volume of a low density black-body radiation with that of an ideal gas. He writes: 'Monochromatic radiation of low density (within the range of validity of Wien's radiation formula) behaves thermodynamically as if it consisted of mutually independent energy quanta of magnitude $R\beta\nu/N$ ' [1, p 97]. Therefore, in Einstein's formula h is replaced by $k_B\beta$, where β turns out to be equal to h/k_B .

The deeply rooted theory of light, with its predictive and explanatory power, constituted a formidable obstacle to the diffusion of Einstein's hypothesis. Hence, it is not surprising that corpuscular treatments of the Doppler effect for light, appeared in the early 1920s, have been ignored by the physics community. Among them, a seminal paper by Schrödinger in which the emission of a light quantum by an atom in flight was treated by writing down the conservation equations of energy and linear momentum for the emitting atom and the emitted quantum [3].

Starting from the 1930s, the two descriptions of light have been merged into quantum electrodynamics: through the quantisation of Maxwell's equations, it was found that the interaction of electromagnetic radiation with charged particles occurs in terms of photons. However, a wide domain of phenomena kept on being adequately described in terms of electromagnetic waves *or* in terms of photons endowed with energy, linear and intrinsic angular momentum. The two descriptions of light are often handled within the so-called wave-particle duality whose popularization is that light sometimes operates as if *constituted* by particles and some other as if *constituted* by waves. We shall postpone the discussion of the epistemological issues involved: for the moment, we shall only acknowledge that many optical phenomena can be *described* either in terms of electromagnetic waves or in terms of photons, without any commitment to how things are (exist) in the World.

In this paper, the two descriptions of light will be considered from two viewpoints: the interference phenomena and the Doppler effect. The historical highlights are *chosen* in order to stress how the corpuscular description of light has emerged and applied to these two phenomena.

For the content and the level, the target of the paper is constituted by high schools and first years university teachers and students.

2. Outline of the paper

In section 3 we deal with the emergence of discrete units of electromagnetic radiation and of light quanta in the first three decades of the 20th century. The ensuing idea of checking if standard diffraction or interference fringes are modified by using low intensity sources is discussed in sections 3.3 and 4, starting from the pioneering experiment of Taylor (1908) up

to the recent one performed with (almost) one photon at a time and with a single photon detector (2005): the wave and photon descriptions of light are evaluated with respect to their capability of predicting the available experimental results. Section 5 recalls that the discrete interaction of light with detectors is shown also by photographs taken with low light intensities. Section 6 deals with the energy fluctuations of black-body radiation in the context of the relation between photon and wave description of light experimentally grounded by single photon interference experiments.

The early steps of the acoustic and luminous Doppler effect are described in section 7, with emphasis on the unified treatment of both effects. Section 7.2 instead, deals with the luminous, relativistic Doppler effect in vacuum treated by Einstein by considering light as constituted by electromagnetic waves (1905). This breakthrough has diverted physicists' attention from the need for a unified, relativistic treatment of the acoustic and luminous Doppler effect. Section 7.3 is entirely dedicated to the retrieval of this unified treatment, begun around the 1980s: this retrieval has conceptual and pedagogical relevance. Section 7.4 presents the photon treatment of the Doppler effect for light. Beside the paper by Schrödinger based on energy and linear momentum conservation laws applied to the emission of a photon by an atom in flight (1922), an interesting calculation due to Emden (1921) and based on linear momentum conservation is retrieved and extended to the relativistic case. Finally, in section 8 the epistemological issues are briefly discussed.

3. Discrete units of electromagnetic radiation. Creativity, hesitations and breakthroughs in crucial years: 1905–1929

3.1. A missed crossroad between classical electrodynamics and light quanta

In his paper on special relativity, Einstein derived the relativistic formula for the Doppler effect of light in vacuum, treated as an electromagnetic wave [4, p 160]: this topic will be discussed in detail in section 7.2. Subsequently, Einstein dealt with the transformation equation of the electromagnetic energy contained in a finite volume. The result is commented by Einstein with these words: 'It is noteworthy that the energy and the frequency of a light complex vary with the observer's state of motion according to the same law' [4, p 163].

This result is contained, though hidden, in Maxwell's theory: to unveil it, it is necessary to consider how the electromagnetic field energy changes in passing from one inertial system to another. If $E_0 \sin(2\pi\nu t)$ is the electric field of a monochromatic electromagnetic wave, the energy density averaged over a period is $\langle u \rangle = \varepsilon_0 E_0^2 / 2$. Einstein's result implies that $\langle u \rangle V \propto \nu$. If we assume that light is composed by light quanta, we can write $\langle u \rangle = n h \nu$, where n is the number of quanta contained in a unit volume. Therefore, the light quanta hypothesis yields the same dependence of the energy contained in a finite volume on the frequency as Maxwell's theory does. It is unlikely that Einstein missed this point; but he did not write this relation, in spite of the fact that it would have given another support to his hypothesis of light quanta. To reword Einstein's statement, it is noteworthy that Maxwell electrodynamics, revisited in a special relativity context, is compatible with a fundamental unit of electromagnetic energy. At this crossroad between special relativity and quanta, Einstein refrained from going through³.

³ Abraham Pais has commented Einstein's attitude with these words: 'I believe that the reason Einstein kept the quantum theory apart from relativity theory is that he considered the former to be provisional (as he said already in 1911) while, on the other hand, relativity to him was the revealed truth' [5, p 909].

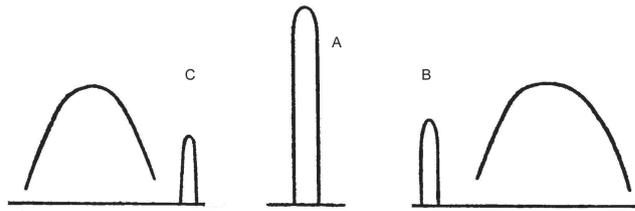


Figure 1. Schematic representation of the Doppler shifts of a Hydrogen line obtained by Stark. On the horizontal axis, the frequency increases to the right; the intensity is reported on the vertical one. See the text. Reproduced with permission from figure 5 of [6]. © AAS.

It must be added that Einstein, after the introduction of the space–time formalism, avoided to use another product of the merging between special relativity and light quanta. The square of the length of the energy–momentum four-vector of a particle is given by:

$$\frac{E^2}{c^2} - p^2 = M^2c^2. \quad (2)$$

If we consider a light quantum as a relativistic particle with zero mass, equation (2) shows immediately that we must attribute to the light quantum a linear momentum $p = h\nu/c$. Einstein arrived at this conclusion only in 1916, within the same approach used in his 1905 paper on light quanta, approach firmly rooted in statistical thermodynamics.

3.2. Stark on light quanta

The opportunity (or necessity) of endowing a light quantum with a linear momentum $(h\nu/c^2)c$ has been instead vindicated by Johannes Stark. In the same year of the appearance of Einstein’s heuristic hypothesis of light quanta, Stark began a series of experiments aimed at measuring the Doppler shift of lines emitted by hydrogen atoms in flight by using canal rays produced in a discharge tube containing hydrogen molecules. Now, we know that the emitting hydrogen atoms were formed behind the cathode of the discharge tube. However, in those times, physicists had no reliable model for the hydrogen atom since Bohr’s model was still to come (1913). This situation is well documented by Stark, when he writes: ‘Is the carrier of the hydrogen series a neutral hydrogen atom or a positively charged hydrogen ion? The canal rays themselves, in hydrogen, consist, according to the measurements of W Wien, of positive hydrogen ions moving with high velocity. If the ions of the canal rays are the sources of the series lines, then it is clear that the series lines must show the Doppler effect. If, however, the series lines have their origin in the neutral atom, then it remains to be explained how the neutral atom acquires the high velocity which the Doppler effect would seem to indicate... It is therefore highly probable that the carriers of the series lines in hydrogen are the positive hydrogen ions’ [6, p 34].

The experiment was far ahead of the theory. Stark observed the emitted lines perpendicularly with respect to the direction of the canal rays (A), facing the canal rays (B) and at 135 degrees with respect to the canal ray direction (C) (figure 1). In all three configurations, Stark observed the un-displaced (normal) line, but its intensity decreased in going from $A \rightarrow B \rightarrow C$. In the perpendicular observation (A) the normal line was alone; in configuration (B), it was accompanied by a broad hazy band on the high energy side, separated from the normal line by an intensity minimum; in configuration (C), it was accompanied by a broad

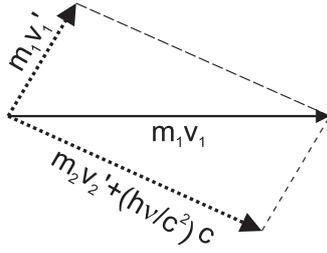


Figure 2. Production of an x-ray quantum. m_1v_1 and m_1v_1' are the momenta of the incident electron before and after the collision; m_2v_2' that of the particle in the anticathode after the emission of the quantum; the particle in the anticathode is considered at rest before emission; $(hv/c^2)c$ is the momentum of the emitted quantum.

hazy band on the low energy side, separated from the normal line by an intensity minimum⁴. Stark interpreted the hazy bands as due to a Doppler blue/red shift of the normal line: the bands being due to the (non thermal) spread of the speeds of the emitting particles. In particular, he wrote for configuration (B):

$$v_{\max} = c \frac{|\Delta\lambda|_{\max}}{\lambda}, \quad (3)$$

where v_{\max} is the maximum speed of the emitting particle and $|\Delta\lambda|_{\max}$ the maximum wavelength shift due to the Doppler effect. From the experimental value of $|\Delta\lambda|_{\max}$ he was able to obtain v_{\max} , which turned out to be of the order of $5 \times 10^5 \text{ m s}^{-1}$ i.e. about 10^2 times larger than the root mean square velocity of atomic hydrogen at room temperature ($2.72 \times 10^3 \text{ ms}^{-1}$).

In 1909, in spite of having attended the meeting of the *Gesellschaft Deutscher Naturforscher und Ärzte* in Salzburg in which the light quantum hypothesis had been vigorously challenged, Stark claimed that an accelerated electron does not emit a spherical wave but directional quanta of energy $h\nu$ [7]. This claim was suggested by the observation that x-rays emitted by an anticathode were able to extract electrons from a metal plate, situated far away from the anticathode, with about the same kinetic energy of the electrons that have generated the x-rays. This behaviour is at odds with the fact that the intensity of an electromagnetic wave goes as the inverse square of the distance from the emitting centre. Stark illustrated the production of an x-ray quantum with the conservation equation of the momenta of the particles involved:

$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1' + m_2\vec{v}_2' + \frac{h\nu}{c^2}\vec{c}, \quad (4)$$

where the subscript 1 refers to the incident electron, the subscript 2 to a particle in the anticathode, the unprimed quantities are those before the emission and the primed ones those after the emission of the X quantum. It is to be noticed that the conservation law is written using newtonian mechanics and that the momentum of the quantum is written with the term

⁴ The presence of an un-displaced line in all three configurations suggests that (almost) thermalized hydrogen atoms were present in the discharge tube behind the cathode. Stark tried to explain the presence of these two minima by supposing that the positive charged canal rays transfer part of their kinetic energy E_{kin} to hydrogen molecules and that this transferred energy is used to set in motion the electrons of the molecules. Since, according to Stark, the emitted electromagnetic radiation is due to the oscillations of the electrons, by writing $\alpha E_{\text{kin}} = h\nu$, where α is a parameter typical of the molecule, he got a minimum value of the canal rays velocity necessary for the emission of the Doppler displaced line.

$h\nu/c^2$ which has the dimension of a mass. Stark stressed that equation (4) can be read not only from left to right but also from right to left: in the former case mechanical energy is converted into electromagnetic energy; in the latter, electromagnetic energy is converted into mechanical energy. Later on, Stark considered a simplified situation, in which the x-ray quantum is emitted along the direction of the momentum of the particle in the anticathode after the emission (figure 2).

After having acknowledged that the speed distribution of the particle in the anticathode after the emission is unknown, Stark considered the case in which the momentum of the particle in the anticathode after the emission is negligible with respect to that of the emitted quantum. In a nowadays description, this situation corresponds to the inelastic scattering of the incident electron by a very massive nucleus with the production of a photon.

Stark's idea about the quantised emission of electromagnetic radiation by a decelerating electron was too ahead of his times. Sommerfeld challenged Stark's proposition by developing his theory of Bremsstrahlung: Sommerfeld's treatment, based on classical electrodynamics, prevailed. With hindsight, we might observe that Stark's idea of using conservation laws for the interaction of an electromagnetic quantum with a particle contains the germ of the explanation by Compton and Debye of the inelastic scattering of x-rays. Reliable experimental evidence of inelastic scattering of γ and x-rays was available around 1915 [8]. Compton himself struggled several years searching for an explanation in terms of classical electrodynamics. Compton developed quantitatively the idea of a big electron: the inelastic scattering 'might be the result of interference between the rays scattered by different parts of the electron, if the electron's diameter is comparable with the wavelength of the radiation' [9, p 484]. Eventually, Compton surrendered and switched to the quantum description suggested some years before by Stark but rejected by the physicists' community: Compton does not cite Stark's papers.

3.3. The discrete units of electromagnetic radiation of Joseph John Thomson

Around 1907, Joseph John Thomson, on the basis of experiments of ionisation of gases with ultraviolet light or x-rays, resumed his idea that the energy of an electromagnetic wave might not be distributed continuously in space but divided in 'units' no longer divisible into smaller parts [10]. This hypothesis was conceptually and epistemologically very different from Einstein's light quanta. According to Thomson, it is possible to conciliate Maxwell's theory with a discrete distribution of electromagnetic energy: actually, he tried to develop a version of Maxwell's theory in which the energy was concentrated in 'electric tubes'. Since these tubes do not occupy the entire space, the distribution of the electromagnetic energy must be necessarily discontinuous. Instead, Einstein conceived his quanta concept as alternative to Maxwell's theory whose validity was confined, when tested in experiments, to values averaged over time of the physical quantities involved. Einstein states clearly which is, in his view, the application domain of classical electrodynamics and which are the phenomena that might not be adequately described by Maxwell's theory. Another basic difference between Einstein's and Thomson's hypotheses lies in the fact that while Einstein wrote the fundamental equation $E = h\nu$ giving the value of the elementary energy unit of an electromagnetic radiation of frequency ν , Thomson never specified the value of the supposed elementary unit of electromagnetic energy. This limit of Thomson's approach clipped the wings of the interpretation of a diffraction experiment, the first one of this kind in the history of physics.

Thomson suggested to Geoffrey Ingram Taylor (one of his students) to do a diffraction experiment with a very low intensity source of light. The idea was that of checking if the supposed discrete nature of electromagnetic energy changes the structure or affects the

visibility of diffraction fringes predicted by the wave theory. In Taylor's experiment, the source was a narrow slit illuminated by a gas flame whose intensity was reduced by smoked glass screens, the diffraction element a needle and the detector a photographic plate. 'Five diffraction photographs were then taken, the first with direct light and the others with the various screens inserted between the gas flame and the slit. The time of exposure for the first photograph was obtained by trial, a certain standard of blackness being attained by the plate when fully developed. The times of exposure were taken from the first in the inverse ratio of the corresponding intensities. The longest time was 2000 hours or about 3 months. In no case was there any diminution in the sharpness of the pattern although the plates did not all reach the standard blackness of the first photograph [11].' According to Taylor, the intensity of the light in the longest exposition was $I = 5 \times 10^{-6} \text{ erg s}^{-1} \text{ cm}^{-2}$ corresponding to an amount of energy per cubic centimeter of $1.6 \times 10^{-16} \text{ erg}$. The paper ends with this statement: 'According to Sir J J Thomson this value sets an upper limit to the amount of energy contained in one of the indivisible units mentioned above'. No comment is made about possible deeper physical implications of the experimental results. Indeed, adopting Einstein's point of view, one could have reasoned as follows. If, in order to maximise the density of light quanta, we assume for the light quantum energy the one corresponding to the longest wavelength of visible light 700 nm, we find that the mean number of quanta contained in a volume of base 1 cm^2 and height 100 cm will be about 6.6×10^{-3} . Of course, this number must be interpreted as the average number of light quanta contained in the specified volume. Therefore, Taylor's experiment might have suggested that diffraction fringes are not modified even when, on the average, less than one elementary unit of electromagnetic energy is in flight in the experimental apparatus.

Taylor's experiment has been the first of a series performed throughout the past century; in all of them the average number of quanta in flight in the apparatus was lesser than one. However, this does not imply that these experiments have been performed with 'one photon at a time'. Furthermore, as stressed below, the photographic plates used in some of these experiments need more than one photon to be triggered.

The basic sensitive unit of a photographic plate is a crystallite of, say, AgBr (size of a few tenths of $1 \mu\text{m}$). Silver bromide is a semiconductor. When a photon is absorbed by a crystallite, an electron is freed into its conduction band; this electron may be trapped by an interstitial silver ion (Frenkel defect) which becomes a neutral silver atom. In order to being 'detected' by the developer, the crystallite must contain more than about four neighbouring silver atoms: hence, the crystallite must absorb a greater number of photons (we must take into account also the efficiency of the entire process). Moreover, the process of formation of the cluster of silver atoms must take place within about one second, which is the lifetime of the isolated interstitial silver atom. A photographic plate is far from being a 'single photon detector'.

3.4. The second occurrence of h and the linear momentum of a light quantum

In 1916–17, Einstein came back to the quantum theory of radiation by trying to develop a new derivation of Planck's radiation law [12]. He considered a cavity containing molecules and electromagnetic radiation in thermal equilibrium. The energy levels of the molecules were quantised. Einstein considered two levels, E_m and E_n with $E_m > E_n$. For the energy density in the cavity, he obtained the formula:

$$u(\nu, T) = \frac{\alpha \nu^3}{e^{h\nu/k_B T} - 1}, \quad (5)$$

where α and h are two universal constants. However, this is not yet Planck's formula since this derivation leaves without expression the value of the universal constant α . If we impose the condition that for $h\nu \ll k_B T$ equation (5) must coincide with the classical (Rayleigh) formula, we get $\alpha = 8\pi h/c^3$ and, at the end:

$$u(\nu, T) = \frac{8\pi\nu^2}{c^3} \frac{h\nu}{e^{h\nu/k_B T} - 1} \quad (6)$$

i.e. Planck's law. Einstein's way to Planck's law is based on a cavity model which takes into account the novelty introduced by Bohr: the emission of electromagnetic radiation by an atom is not due to the oscillations of the electrons but to a 'quantum jump' from an energy level to a lower one. Planck supposed that the equilibrium energy pertaining to the resonators (electrically charged oscillators) of frequency ν must be attributed in terms of a basic unit $h\nu$ where h is a universal constant: otherwise, the entropy of the resonators would become infinite. The discreteness of the energy emerges as a sufficient condition so that the resonators' entropy remain finite. Instead, in Einstein's derivation, the discreteness of the energy is supposed from the beginnings. As a direct product of the derivation, the constant h , emerges within Bohr's transition formula between two atomic energy levels: $E_m - E_n = h\nu$.

In the last part of the paper, Einstein demonstrated that, if a molecule, in the emission/absorption of a light quantum, exchanges not only an energy $h\nu$ but also a linear momentum $h\nu/c$, then the molecules speed distribution is that demanded by Maxwell's distribution law. Einstein commented the importance of the linear momentum exchange with these words: 'But in *theoretical* investigations these small effects are definitely as important as the more prominently appearing *energy* transfers by radiation, because energy and momenta are always intimately linked together. Therefore, a theory can only be viewed as justified if it shows that the momenta that are transferred from radiation onto matter lead to motions as they are demanded by the theory of heat [12, p 233]' (italics in the original text).

Strangely enough, Einstein himself ignored this warning. In 1921 he published a paper in which was proposed an experiment for discriminating between the two theories of light. In doing so, he stated that '... if one considers Bohr's emission condition

$$E_2 - E_1 = h\nu$$

that connects the energy change of the atom with the emitted frequency, which is so fundamental in quantum theory, then one is inclined to assign a unique frequency to every elementary emission process, and also to the process of emission from a moving atom' [13, p 255]. This 'inclination' is at odds with what stated by Einstein in his 1916 paper [12]: when an atom emits a light quantum not only its energy, but also its linear momentum changes.

In fact, one year later, Robert Emden showed how the classical (non relativistic) formula for the Doppler effect can be derived by describing light as constituted by light quanta endowed with linear momentum $h\nu/c$ [14]. In section 7.4.1 Emden's treatment is discussed and extended to the relativistic case.

3.5. Conservation of energy and linear momentum

In 1922, Schrödinger showed that the Doppler effect can be derived by writing down the relativistic conservation equations for energy and linear momentum when an atom emits a light quantum [3] (section 7.4.2).

In 1923–24, Compton [9] and Debye [15] explained the inelastic scattering of x-rays by applying the conservation laws of energy and linear momentum to the scattering of a x-ray quantum by an electron considered at rest. The conceptual approach was the same as that of Stark (section 3.2): the difference being the use of relativistic mechanics and the conservation law of both energy and momentum. This accomplishment is usually considered as a trigger for the change of physical community's attitude toward light quanta.

3.6. *The intrinsic angular momentum of a photon*

In 1929 Pokrowsky suggested that if we consider a beam of circularly polarised light as constituted by photons, then we should attribute to each photon an angular (intrinsic) momentum equal to $\pm\hbar$ along the direction of propagation of the photon [16]. In 1936, Beth obtained experimental evidence that circularly polarised light passing through a half-wave quartz plate transmits to the plate an angular momentum equivalent to $2\hbar$ per photon (a right circularly polarised photon comes out of the half-wave plate left circularly polarised) [17]. In the late 1940s, Carrara obtained similar results with microwaves [18, 19]: at a given radiation intensity, the photon flux varies as $1/\omega$; therefore, the use of microwaves instead of visible radiation enhances the torque effect (it is proportional to the photon flux). An experiment conceptually similar to that of Beth has been carried out recently in more sophisticated and controlled experimental conditions [20]. The effect of photons' intrinsic angular momentum on microscopic particles has also been studied: see, for instance, [21].

4. Light quanta and interference

4.1. *Refining Taylor's 1909 experiment in a changed conceptual context*

As we have seen in section 3.3, the hypothesis of a discrete unit of electromagnetic radiation has prompted since the beginnings the idea of checking if diffraction or interference experiments would be affected in some way when the light source is so faint that the supposed discrete constitution of light might show its effects. Taylor's diffraction experiment with faint light has been the first one of this kind: its poorly controlled parameters cannot overshadow its conceptual relevance and its historical primacy.

When in 1927 Dempster and Batho performed another interference experiment with a low intensity light source [22], the moods of the physicists community were profoundly changed with respect to the first years of the century. The title of their paper, 'Light quanta and interference', unthinkable at Taylor's times, reflects the change occurred in the experimental and theoretical background from the 'homemade' experiment of Taylor. Dempster and Batho set up a sophisticated apparatus: the light source was monochromatic (the line at 447.1 nm of helium), its intensity was measured by comparison with the corresponding emission of a blackbody at 1125 K: only the detector was of the same type, i.e. a photographic plate. They made two experiments: one with an echelon diffraction grating and another with an air film between two parallel glass plates. The concept of light quantum adopted by Dempster and Batho has two properties: it is a discrete element of energy *but* it is sufficiently spread in order to cover an area comparable with that of the effective section of the diffraction instrument. This implies that their quantum splits between the two possible paths of the interference set up. Actually, Dempster and Batho interpreted the fact that the interference patterns were undistinguishable from those obtained with a standard intense source as a corroboration of the hypothesis that the light quantum spreads over a finite area: they talk of 'wavefront covered by a single light quantum'. However, since in their measurements there

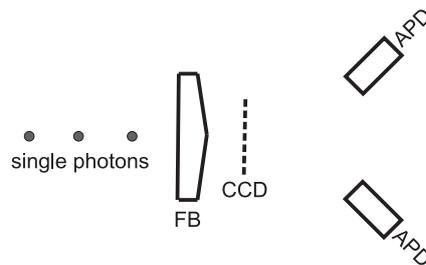


Figure 3. FB is a Fresnel biprism; CCD an intensified CCD camera operated at $-25\text{ }^{\circ}\text{C}$; APD are two silicon avalanche photodiodes operated in photon counting regime. Coincidence measurements are performed by removing the CCD camera.

was, on the average (like in Taylor's case), less than one quantum at a time in flight in the apparatus, they cautiously do not come to the conclusion that a single quantum produces a full interference pattern, though so faint as to be undetectable.

4.2. Photons interference and first 'which path measurement'

After the second world war, this type of experiments has been resumed again in the mid 1950s by Jánossy and Náray [23]. The interference apparatus was a Michelson interferometer, the spectroscopic line was mercury's green line at 546.1 nm, the detector a photomultiplier tube. The optical arrangement produced the interference pattern on a screen with a narrow slit: the screen and the photomultiplier could be moved together along a direction perpendicular to the slit so that to measure the light intensity at different points of the interference pattern. Owing to the low intensity of the source, there was, on the average, less than one photon at a time in flight in the apparatus. The interference pattern at low light intensity was indistinguishable from that at high intensity. In a previous work, Adám, Jánossy and Varga, had found (for the first time), by coincidence measurements with two photomultipliers, that if the light coming from a faint source is divided by a beam splitter, there is no correlation between the signals of the two photomultipliers, thus supporting the idea that a photon cannot be divided into two parts [24]. The reliability of these coincidence measurements has been challenged by Brannen and Ferguson [25], who however, confirmed the results of the Hungarian researchers⁵. Jánossy and Náray' problematically wrote: 'Thus each photon is clearly under the influence of both mirrors and the existence of the interference pattern at low intensities seems to show that after all each photon splits into two halves, these halves are reunited later and give a beam with interference maxima and minima. This conclusion seems to contradict that drawn from the coincidence experiment according to which the photons do not split' [23, p 406].

The model of light quantum adopted by Dempster and Batho and by Jánossy and Náray differs substantially from Einstein's concept proposed in 1905. While Einstein held that light quanta 'are localised in points in space, move without dividing, and can be absorbed or generated only as a whole', Dempster and Batho and Jánossy and Náray supposed that the quantum can be divided by an interference apparatus as it happens for a light wave. Furthermore, Dempster and Batho supposed that the photon is enough spatially extended so that to cover a critical part of the interference apparatus.

⁵ Brannen and Ferguson stressed that, in order to be sure that we have observed no correlation, we must firstly prove that our apparatus is able to detect correlated photomultiplier signals. The Hungarians did not do this check.

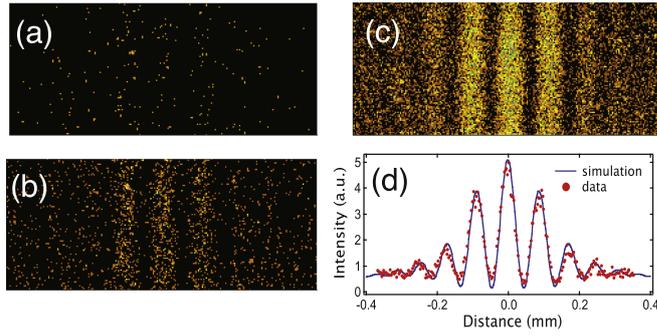


Figure 4. The interference pattern is built up on the camera screen spot by spot: (a) 272 photons; 2240 photons produce a visible interference pattern (b); 19 773 photons yield a well defined one (c). Figure (d) shows that the experimental pattern corresponding to about 20000 photons is well interpolated by classical electrodynamics. Figure reproduced from figure 4 of [29] with kind permission of The European Physical Journal (EPJ).

Subsequently, several other experiments have been carried out [26–28]. However, we shall discuss only the one performed in the centenary of the ‘heuristic hypothesis’ of light quanta [29]. Our choice does not reside in the centenary coincidence but on the fact that the experiment has been carried out with a controlled source which emits (almost) one photon at a time *and* with a single photon detector.

4.3. Single photon interference

The experimental setup is illustrated in figure 3.

The features of the experiment are:

- i. The light source emits (almost) one photon at a time, when suitably excited in a controlled way. The light source is a nitrogen-vacancy colour centre in a diamond nanocrystal.
- ii. The diffraction element is a Fresnel prism that is equivalent to a double slit set up.
- iii. The interference pattern is observed on the screen of an intensified CCD camera (*iStar* from Andor Technologies, cooled at -25 °C), placed in the region of overlapping interference paths.
- iv. By removing the CCD camera, it is possible to demonstrate single photon behaviour by the absence of time coincidence between photon detection along the two paths: each photon goes through one or the other interference path, without dividing between them.

Point (iv) has been dealt in the following way. If one describes light as a wave and assumes that the photon detector is quantised, then it is shown that the number of coincidences N_C must satisfy the inequality:

$$\alpha = \frac{N_C N_T}{N_1 N_2} \geq 1, \quad (7)$$

where N_T is the number of trigger pulses applied to the emitter and N_1 , N_2 the photon counts along paths 1, 2 respectively. For single photon behaviour, $\alpha = 0$; the experimental value was 0.13 ± 0.01 . According to the authors, this discrepancy ‘is due to remaining background fluorescence of the sample and to the two-phonon Raman scattering line, which induces a

non-vanishing probability of having more than one photon in the emitted light pulse'. Therefore, within this limit, point (i) is satisfied. Furthermore, if a faint laser pulse (mean number of photon per pulse below 10^{-2}) is used, it turns out that the correlation parameter α is about unity, thus proving, as already pointed out above, that experiments carried out with faint light are not single photon experiments.

In order to understand the meaning of the patterns obtained, it is necessary to keep in mind how an intensified CCD camera works. A photon emitted by the source falls on the photocathode of the intensifier; the emitted photoelectron, accelerated through a micro-channel-plate produces at least 10^4 electrons that, falling on a phosphor screen, produces photons; finally, these photons are collected to the CCD of the camera by a fibre optic coupler or by a lens. Then, the original single photon emitted by the source produces at the end a bright spot on the camera screen.

The interference patterns are shown in figure 4. Each interference pattern is built up on the camera screen spot by spot: about 2000 photons detected by the intensifier are sufficient to produce a visible interference pattern (figure 4(b)); about 20 000 photons yield a well defined one (figure 4(c)). Figure 4(d) shows that the experimental pattern corresponding to about 20 000 photons is well interpolated by classical electrodynamics.

Coincidence measurements performed by removing the CCD camera show that each photon goes through one or the other interference path, i.e through one or the other 'slit' (in the limit in which the Fresnel biprism is equivalent to a two slits setup). Of course, in this case, we cannot see the interference fringes because, for making the coincidence measurement we must remove the CCD camera. Finally, this experiment shows that predictions of classical electrodynamics agree with experiment when the number of photons is sufficiently high, no matter if used one at a time or all together in an intense flash. This statement implies that Maxwell's theory can predict the probability that a photon will reach a point on the detector if it is assumed that this probability is proportional to the classical intensity predicted for that point. Of course, the idea of a discrete constitution of electromagnetic radiation is extraneous to Maxwell's theory because it uses continuous spatial functions. However, light quanta might be compatible with Maxwell's theory as well as it is the discrete nature of electricity. In the case of electricity, it is sufficient to define the charge density ρ as $\Delta q/\Delta V$ where the volume element ΔV is sufficiently large in order to avoid abrupt spatial changes in the charge density and sufficiently small in order to keep ρ sensitive to spatial variations of the charge distribution. In the case of light quanta, it is sufficient that the number of light quanta used is large enough, notwithstanding if used one at a time or all together.

After all, we should perhaps recognise that Thomson's idea of reconciling Maxwell's theory with a discrete constitution of electromagnetic energy was not completely unsound. Anyway, we must acknowledge that Maxwell's theory has survived the advent of the fundamental unit of charge, the disappearance of the ether and the advent of the light quanta hypothesis. Furthermore, the quantisation of its equation in vacuum has given birth to quantum electrodynamics. Probably, there is no other physical theory that has proven to be so durable.

4.4. Single photon interference: a quantum theoretical description

The theoretical treatment of double slit interference by using the wave (electromagnetic) description of light is a basic item of physics courses. Also in quantum physics the double slit

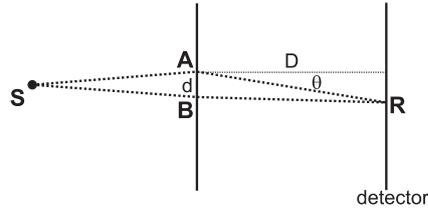


Figure 5. Two slits interference. S is the source, A and B the two punctiform slits. The distance D is supposed much larger than the distance d between the two slits.

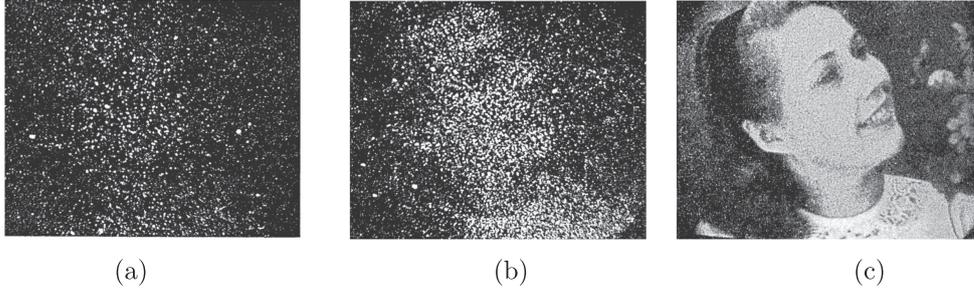


Figure 6. Photographs with faint light: (a) 1×10^3 photons; (b) 1.2×10^4 photons; (c) 2.8×10^7 photons. Pictures reproduced from [30] by permission of the OSA.

experiment is considered a fundamental application of the superposition rule due to the linearity of Schrödinger's equation. Following Feynman [31, p 37–5], let us consider an idealised double slit experiment: the slits are punctiform (figure 5).

The probability to find a particle at point R on the detector when only slit i , ($i = A, B$) is open can be written as:

$$P_i(R) = |\Psi_i|^2 = |C_i e^{i\varphi_i}|^2 = C_i^2, \quad (8)$$

where Ψ_i is a probability amplitude, $\varphi_i = 2\pi l_i/\lambda$ with λ given by the de Broglie wavelength of the particle and l_i is the length of the path. Instead, when both slits are open, by putting $C_A = C_B = C$ (which amounts to suppose that the two slits are perfectly symmetric in the experimental setup), we have:

$$P_{AB}(R) = |\Psi_A + \Psi_B|^2 = 2C^2 + 2C^2 \cos \delta = 4C^2 \cos^2 \left(\frac{\delta}{2} \right), \quad (9)$$

where $\delta = \varphi_A - \varphi_B$. Since $D \gg d$, $\delta = 2\pi d \sin \theta/\lambda$. Therefore:

$$P_{AB}(R) = 4C^2 \cos^2 \left(\frac{\pi d \sin \theta}{\lambda} \right). \quad (10)$$

In the case of monochromatic photons, λ is the wavelength of the electromagnetic wave associated to them through the de Broglie relation $\lambda = h/p = h/(h\nu/c) = c/\nu$.

The formula derived by supposing that light is constituted by electromagnetic waves is (when the dimensions of the slits are finite):

Table 1. Classical and quantum description of light interference have the same mathematical structure.

Waves	Photons
$\lambda = c/\nu$	$\lambda = h/p = c/\nu$
Electric field \vec{E} $\vec{E} = \vec{E}_1 + \vec{E}_2$	Probability amplitude $\Psi = Ce^{i\varphi}$ $\Psi = \Psi_1 + \Psi_2$
Energy density $\propto E^2$	Probability $\propto \Psi ^2$

$$I = 4I_0 \cos^2 \left(\frac{\pi d \sin \theta}{\lambda} \right) \left[\frac{\sin(\pi b \sin \theta / \lambda)}{\pi b \sin \theta / \lambda} \right]^2, \quad (11)$$

where I_0 is the maximum intensity on the detector when only one slit is open and b is the slits' width. If $b \rightarrow 0$, the expression under square brackets tends to 1 and the two formulas (10), (11) assume the same form. However, while (11) deals with waves intensities, (10) refers to probabilities. The same mathematical structure of the wave and photon treatment of interference is illustrated in table 1.

This common mathematical structure explains why the wave description predicts the correct result when the number of photons is statistically significant, independently from the fact that they are used one at a time or all together. However, the same common mathematical structure does not tell us why a statistically significant number of photons can be treated as a wave, when they are used all together.

In the photons description of interference we cannot say which slit the photon has passed through. Similarly, the wave description cannot say which is the path of the energy between the two slits and the detector: this is another aspect of the common mathematical structure of the two descriptions.

The quantum description of interference uses, in its derivation, de Broglie's wavelength, i.e. a physical quantity typical of a wave. This feature reflects the fact that a 'particle' is described in quantum physics by a set of physical quantities which include de Broglie's wavelength. For instance, an electron, beside classical properties like mass and charge and linear momentum is endowed with de Broglie's wavelength. This mixture of particle and wave properties has suggested to Mario Bunge to propose the label 'quanton' for theoretical terms (theoretical entities) like electron, atom, photon... whose behaviour is described by quantum physics [32, p 270].

As shown in table 1, the superposition rule (9) has its exact counterpart in classical electrodynamics where the superposition of electric fields is dictated by the linearity of Maxwell equations. Rather recently, the simple superposition rule (9) used for describing two slits interference experiments has been challenged [33–35]. The basic objection can be illustrated in this way. The standard superposition rule presupposes implicitly that the probability amplitude $\Psi_A(R)$ is not affected by the fact that slit B is closed or open. It is held that this assumption may not be correct. As a matter of fact, when both slits are open, a possible path for reaching point R on the detector may pass once through both slits and two times through one of them: similar paths are called non classical ones. The possible contribution of non classical paths might be revealed in a three slits experiment [34, 35]. According to [35], in a three slits experiment a parameter κ taken as a measure of the contribution of

non classical paths will be of the order of 10^{-6} if the experimental parameters used in the experiment performed by [36] are taken into account. Unfortunately, the experimental detection limit for κ in [36] is 10^{-2} . Hence, an experimental check of non classical paths contribution to interference is still to come.

5. Photographs with faint light

The discrete behaviour of light has been evidenced also by making photographs with faint light (figure 6). As in the case of interference measurements the picture is made up of spots: only when their number is sufficiently high, the picture of the woman is distinguishable. As in the case of interference, when the number of photon used is sufficiently high, the production of the picture can be described by the wave theory of light.

Interference and photographs are not the only physical processes which show that, when the number of photons is high enough, light can be described as an electromagnetic waves. Indeed, rather recently, the electric field of a short (few cycles) but intense laser pulse has been measured [37].

6. Ambiguous fluctuations

The double slit single photon interference experiment (section 4.3) shows that the interference fringes are correctly described by Maxwell's theory when the numbers of photons used is high enough, regardless if they are used one at a time or all together. This experiment, which establishes a correlation between the wave and the photon description of light, suggests to revisit a paper by Einstein on the energy fluctuations of the black-body radiation [38]. Obviously, in this case, we are dealing with photons that are contemporaneously at work. Einstein showed that the energy fluctuation $\overline{\varepsilon^2}$ of the radiation whose frequency lies in the interval between ν and $\nu + d\nu$ is given by (again, Einstein did not use the constant h):

$$\begin{aligned}\overline{\varepsilon^2} &= \overline{(E - \bar{E})^2} = \overline{E^2} - \bar{E}^2 \\ &= \left(u(\nu, T)h\nu + \frac{c^3}{8\pi\nu^2}u^2(\nu, T) \right) \nu d\nu,\end{aligned}\quad (12)$$

where E is the energy of the radiation with frequency ν , $u(\nu, T)$ the energy density (per unit frequency and unit volume) of the radiation and ν the volume occupied by the radiation.

It is instructive to rewrite equation (12) so that it contains only an average number of photons. Planck's formula can be written as:

$$u(\nu, T) = Z(\nu)[\bar{n}(\nu, T)h\nu], \quad (13)$$

where

$$Z(\nu) = \frac{8\pi\nu^2}{c^3}, \quad \bar{n}(\nu, T) = \frac{1}{e^{h\nu/k_B T} - 1}. \quad (14)$$

We can denote $Z(\nu)$ (a number per unit volume and unit frequency) with the word 'acceptors', since they can accommodate energy quanta. In Planck's derivation, $Z(\nu)$ is the density of 'resonators' [39]; in Debye's, the density of normal modes of vibrations of stationary electromagnetic waves [40]; in Bose's, the density of cells of volume h^3 in the phase space [41]. $\bar{n}(\nu, T)$ is the average number of photons that, at thermal equilibrium, is

attributed to each acceptor. All three derivations, use the same formula (or an equivalent formula) which yields the number of ways R of distributing P indistinguishable energy quanta among N distinguishable acceptors:

$$R = \frac{(N + P - 1)!}{(N - 1)!P!}. \quad (15)$$

Planck's and Debye's derivations do not escape Einstein's objection: since $Z(\nu)$ is obtained within Maxwell's theory which uses continuous spatial functions, its use is incompatible with the quantisation of the radiation energy [42]. This is the reason why Einstein considered Planck's formula in agreement with experiment, but flawed by a conceptual contradiction. For the same reason, Einstein looked for a new derivation of Planck's law (section 3.4) and welcomed Bose's paper by favoring its publication in the *Zeitschrift fur Physik* after its rejection by the *Philosophical Magazine* [43]. In the following, we shall move within Bose's derivation of Planck's formula that deals only with photons.

Then, using equations (13) and (14), equation (12) assumes the form:

$$\overline{\varepsilon^2} = Z(\nu)(h\nu)^2\bar{n}(\nu, T)[1 + \bar{n}(\nu, T)](v d\nu). \quad (16)$$

Formula (16) contains only the photons energy $h\nu$ and their average number $\bar{n}(\nu, T)$: the only reminiscence of waves is due to the basic relation $E_{\text{ph}} = h\nu$. In equation (16) there is not any wave-particle duality, whatever this duality might mean. The energy fluctuation is given by the sum of two terms, one proportional to the average number of photons $\bar{n}(\nu, T)$, the other proportional to its square. The former is typical of a system of independent 'particles' of energy $h\nu$, like the molecules of a classical ideal gas. The presence of the other term says that the statistics of photons of a black-body radiation is *not* like that of the molecules of a classical ideal gas. In fact, it obeys Bose's statistics (in which 'particles' are indistinguishable), not Boltzmann's⁶.

Einstein observed that dimensional considerations suggest that the term proportional to \bar{n}^2 (written in Einstein's form) is typical of waves. Einstein's interpretation of equations (12) (and (16)) is supported also by the following arguments. If, in the calculation of the energy fluctuation $\overline{\varepsilon^2}$, we start from Wien's approximation of Planck's formula, valid for $h\nu \gg k_B T$ and used by Einstein for his heuristic proposal of light quanta, we get only the 'corpuscular' term proportional to the average number of photons \bar{n} . On the other hand, if we start from the classical Rayleigh-Jeans's approximation [44, 45], valid for $h\nu \ll k_B T$, we get only the wave term, proportional to the squared average number of photons \bar{n}^2 . Rayleigh-Jeans' classical formula can be derived directly by supposing that the radiation is constituted by electromagnetic waves and that, at thermal equilibrium, we must attribute to each normal mode of vibration the energy $k_B T$. Then, the fact that starting from Rayleigh-Jeans' formula we get only the term proportional to the square of the average number of photons support the view that this term is typical of waves. The formal proof that the term proportional to \bar{n}^2 (written in Einstein's form) is typical of electromagnetic waves has been given in 1912 by Lorentz [46].

There is a correspondence between the single photon interference patterns and the energy fluctuations of the black-body radiation. In both cases, the critical parameter is the number of photons that take part to the phenomenon. With some specifications. In the case of

⁶ If particles are distinguishable, the interchange of two of them yields a new state of the system of particles (Boltzmann). If they are not, the interchange of two particles yields the same state of the system (Bose). Quantum mechanics holds that there are only two possible wavefunctions for a system of particles when two of them are interchanged: symmetrical, for 'bosons' and antisymmetrical for 'fermions'. Of course, this was not known when Bose wrote his paper. Abraham Pais has commented by saying: 'I believe there has been no such successful shot in the dark since Planck introduced the quantum in 1900' [5, p 895].

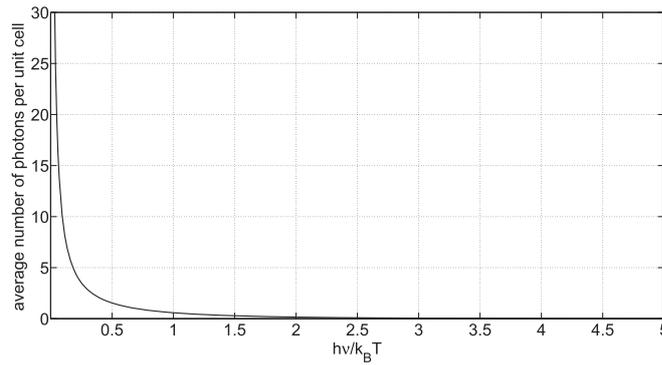


Figure 7. Average number of photons $\bar{n} = 1/(e^{hv/k_B T} - 1)$ attributed to each cell of volume h^3 at thermal equilibrium.

interference, the ‘number of photons’ is simply the number photons used, regardless if they are used one at a time or all together. In the case of energy fluctuations of black-body radiation, the ‘number of photons’ is the average numbers of photons attributed at thermal equilibrium to each ‘acceptor’, i.e. to each cell of volume h^3 of phase space. When the ‘number of photon’ is small, the interference pattern is an assemblage of spots while the energy fluctuation of the black-body radiation is that typical of particles. When the ‘number of photon’ is large, the interference pattern is constituted by continuous fringes described by Maxwell’s theory, while the energy fluctuation of the black-body radiation is that typical of waves. In the intermediate case, the interference pattern is still made up of spots, but the interference fringes begin to appear; correspondingly, the energy fluctuation of the black-body radiation contains both terms, corpuscular-like and wave-like. It is to be remembered that ‘small’ and ‘large’ correspond to different values in the two cases. Figure 7 illustrates the case of the energy fluctuation of black-body radiation: this figure corresponds to figures 4(a)–(c) of interference.

Historically, Einstein’s 1909 paper has been considered as the source of the so-called wave–particle duality. In retrospect, we might say that the debate on this issue has been more a battle field of philosophical stands than the occasion for clarifying a fundamental physical question. As Levy-Léblond puts it: ‘To classical or post-classical physicists there were only two types of physical entities, namely particles and waves. Since the strange quantum beings showed a behaviour reminiscent sometimes of particles and sometimes of waves, it was not unnatural to discuss them in classical terminology, under the philosophical cover of the ad hoc ‘wave–particle duality’ [47]. Then, Levy-Léblond goes on to advocate the use of the word ‘quanton’ proposed by Bunge [32] for the quantum theoretical terms (theoretical entities) described by quantum physics. Certainly, the single photon interference experiment and, even more the interference of ‘particles’ endowed with mass are the best advocates of the use of the theoretical term ‘quanton’. However, the question raised above: ‘why, when the number of photons used, regardless if they are used one at a time or all together, the interference fringes are correctly described by Maxwell’s theory?’ remains still without an answer. Our feeling is that the answer to this question does not concern the reconciliation between a 19th century theory with new experimental acquisitions but, instead, asks for a new insight.

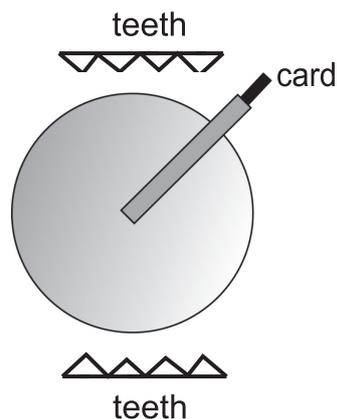


Figure 8. Fizeau's instrument. A cardboard on a rotating wheel strikes the teeth on the concave side of an outer arc.

7. Doppler effect

7.1. The beginnings

The Doppler effect, for acoustical and light waves, has been theoretically predicted by Christian Andreas Doppler [48]. On 25 May 1842 Doppler delivered a lecture to the Royal Bohemian Scientific Society in Prague entitled 'Concerning the coloured light of double stars and of some other heavenly bodies'. The waves (acoustical or luminous) were considered travelling in a medium at rest and their frequencies expressed in terms of the velocity of the emitter or the absorber with respect to the medium. Doppler considered the emitter or the absorber moving along the line of propagation of the wave, in the same or in the opposite direction of the wave. Doppler wrote formulae that can be summarised in a single one:

$$f_a = f_e \frac{V \pm v_a}{V \mp v_e}, \quad (17)$$

where V is the propagation speed of the signal, v_a and v_e the velocities of the absorber and emitter, f_a and f_e the frequencies measured by the absorber and the emitter respectively. In this formula the sign (+) in the numerator corresponds to the motion of the absorber towards the emitter, while the sign (−) corresponds to motion of the absorber away from the emitter. In the denominator, the role of the signs is interchanged.

On 23 December 1848, Fizeau presented at the Societ  Philomathique de Paris a report entitled 'On the effects of motion on the pitch of acoustic vibrations and on the wavelength of luminous rays'. An abstract of this note, without any formula, appeared in the Proceedings of the Society, but the full text was published only in 1870 [49]. Unfortunately, because the report contained a treatment of the 'Doppler effect' for sound which included the case of arbitrary inertial motion of the source and of the absorber, a detailed description of an experiment for sound and a discussion about a possible application of the 'Doppler effect' for light in astronomy. The formula obtained by Fizeau for sound is equivalent to equation (27) of section 7.3. The theoretical treatment was supported by an elegant experiment (figure 8).

Let us suppose that an observer stands on the right side of the figure with her/his hear aligned with the centre of the rotating wheel. If the wheel rotates in the clockwise direction and its rotation speed is conveniently adjusted, the observer will hear a high tone when the

cardboard hurts the teeth at the top and a lower one when the cardboard hurts the teeth at the bottom. This is due to the fact that the vibrating cardboard is in motion towards (top teeth) or away (bottom teeth) from the observer. As for light, Fizeau stresses that, in astronomy, one should look at the shift of the wavelength of each ray and not to the colour of the light. He argues that the ‘Doppler’ shift does not change the colour of light emitted by a luminescent body. In general, this is not true, for two reasons. First of all because the Doppler shift depends on the frequency; secondly, because one should take into account also the fact that the intensity of the emitted light is frequency dependent. However, Fizeau was right in indicating the wavelength of each ray as the physical parameter of major interest. Fizeau’s report remained practically unknown⁷.

The ‘Doppler principle’ (as it was called in those times) sparked a vivacious controversy, particularly focused on the predicted effect on the colours of stars. The way towards the acceptance of the ‘principle’ has been long. For acoustic signals, the Doppler effect has been qualitatively supported in 1845 by placing a horn player on a train passing by musicians with excellent pitch perception [50]. Some years later, Ernst Mach, in addressing the dispute about the ‘principle’, carried out the following experiment [51]. Mach placed a whistle at the end of a tube (183 cm long) rotating about an axis passing through its centre; the whistle was blown by the wind forced along the axis of the tube⁸. In Lord Rayleigh’s words: ‘An observer situated in the plane of rotation hears a note of fluctuating pitch, but if he places himself along the axis of rotation, the sound becomes steady’ [53, p 155].

The case of light has been much more troubled. Initially, the debate has been diverted by Doppler’s stress on the visual colour of stars, instead of the spectroscopic lines emitted by them. The seminal works of Kirchhoff and Bunsen (1859–1860) gave a decisive impulse to the spectroscopy of lines emitted by chemical elements of stars. However, sufficiently accurate spectroscopic measurements (not relying on subjective visual judgment) had to wait the production and diffusion, in the mid-1880s, of dry gelatin photographic plates. Therefore, only at the end of the 1880s, was Hermann Carl Vogel able to register on photographic plates the Doppler shifts of several lines emitted by stars in motion along the line of light emission [54].

7.2. Doppler effect for light in vacuum described as an electromagnetic wave

In the 19th century, the theoretical treatment of the Doppler effect was the same for acoustic and light waves. For instance, Fizeau derived the formula for acoustic waves and applied it to light because ‘These considerations concerning sound waves can be applied to luminous phenomena, as they are described by the wave theory...’ [49, p 216]. It is to be stressed that in those times light waves were always supposed to propagate in a medium (ether).

⁷ The publication of Fizeau’s paper in 1870 was accompanied by a note in which it is stated that ‘This paper has not been published before because the author has considered up to now as sufficient the abstract, published, at that time, in the Proceedings of the Society. The same paper is published nowadays because many people (including one of the Editors of the Annals), after having recently asked the author for information, considered that the abstract in question was too abridged and that it would be useful to complete it with the present publication which has been made on the old text, without any modification required by the current state of science’. We do not know if the 1848 choice of Fizeau was due to an undervaluation of the value of his paper or to an overvaluation of the diffusion of the Society’s Proceedings. Anyway, this choice has been very likely the basic reason why today, apart from French literature, the ‘strange effect’ (as it was called by Fizeau) is now known worldwide as the ‘Doppler effect’ and not as the ‘Doppler–Fizeau’ effect.

⁸ It is worth noticing that the idea of testing the Doppler effect using rotating sources begun by Fizeau and continued by Mach has been used also in the 1960s for studying the Doppler shift of photons emitted without recoil (Mössbauer effect) in rotating devices [52].

Once again, a turning point has been due to Einstein, who, in his paper on special relativity, derived the relativistic formula for the Doppler effect of light in vacuum, treated as an electromagnetic wave:

$$\nu = \nu' \frac{\sqrt{1 - v^2/c^2}}{1 - (v/c) \cos \theta}. \quad (18)$$

Here, we have implicitly supposed, as usual, that we are dealing with two inertial reference frames O and O' , whose axes are parallel and oriented in the same way: O' is considered in motion, with respect to O , with velocity v along the common $x \equiv x'$ axis. Then, θ is the angle formed by the direction of propagation of the wave with the x axis. The angle θ is connected to the angle θ' through the aberration equation:

$$\cos \theta = \frac{B + \cos \theta'}{1 + B \cos \theta'}; \quad B = \frac{v}{c}. \quad (19)$$

In formula (18), there are two novelties: the medium (ether) has vanished and the relativistic time dilation factor $\sqrt{1 - v^2/c^2}$ has appeared. However, after Einstein's breakthrough, the need for a relativistic treatment of the Doppler effect for both acoustic and luminous signals in material media, has been overlooked. As we shall see in next section, only rather recently has this need been recovered.

Einstein's 1905 derivation uses the transformation equations for the components of the electric field and for the spatial and time coordinates. Einstein obtains equation (18), and, as a by-product, the formula for the aberration of light. As recalled in section 3.1, subsequently, Einstein derived the formula for the transformation of the electromagnetic energy contained in a finite volume $\langle E_{em} \rangle$:

$$\langle E_{em} \rangle = \langle E'_{em} \rangle \frac{\sqrt{1 - v^2/c^2}}{1 - (v/c) \cos \theta}. \quad (20)$$

Formula (20) can be derived easily by using the transformation equation of the energy-momentum four-vector relative to a finite volume of space traversed by an electromagnetic wave. Since:

$$\frac{\langle E'_{em} \rangle}{c} = \gamma \left(\frac{\langle E_{em} \rangle}{c} - B \cos \theta \right) \quad (21)$$

we get immediately equation (20) by solving for $\langle E_{em} \rangle$.

For $\theta = \pi/2$, equation (20) reads:

$$\langle E_{em} \rangle = \langle E'_{em} \rangle \sqrt{1 - v^2/c^2} \quad (22)$$

which can be compared with the transformation equation of the energy of a massive particle at rest in the moving reference frame O' :

$$E = E' \frac{1}{\sqrt{1 - v^2/c^2}}. \quad (23)$$

This difference was boldly used by Louis de Broglie [55] by assuming that a massive particle can be treated as a light quantum, thus arriving at associating a wave to it⁹.

⁹ This bold step has been preceded by several papers where light quanta were endowed with a very small, but not null, mass. Therefore, no surprise that this path has been completed, at last, by treating a massive particle as a light quantum endowed with a (small) mass.

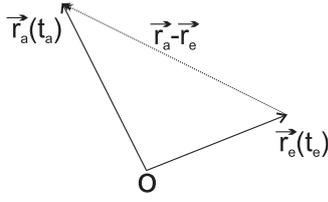


Figure 9. At the instant $t = 0$ both emitter and absorber are at the origin O of the laboratory's frame of reference: the medium-homogeneous and isotropic-is at rest in the laboratory reference frame. See the text.

7.3. General equations for acoustic and luminous Doppler effect

The derivation by Einstein of the relativistic formula for the luminous Doppler effect in vacuum produced the side effect of forgetting the habit, typical of the 19th century, of a unified treatment of the acoustic and luminous Doppler effect. Nowadays, it is hard to find such a unified treatment in a contemporary textbook. However, in the 1980s, the problem of a general relativistic treatment of the Doppler effect for acoustic and light signals had been dealt with by several authors in various journals: see, for instance [56, 57], and references therein. We shall develop the derivation in two steps: firstly, the calculation will be performed in the inertial laboratory reference frame; then, by switching to the reference frames co-moving with the emitter and the absorber, the time dilation factors will be taken into account. The first step yields the classical formula; the second step the relativistic one. See also [56, 58].

We consider an emitter of acoustic or light signals and an absorber in arbitrary inertial motion with respect to the medium of propagation of the signals. It is convenient to describe the phenomenon in the inertial frame with origin in O with respect to whom the medium-homogeneous and isotropic-is at rest (the laboratory frame). Owing to the homogeneity of time and the homogeneity and isotropy of the medium, we can suppose, without any loss of generality, that the emitter and the absorber are at the origin O at the instant $t = 0$ (figure 9). If at the instant $t = 0$ the emitter sends a signal of ideally null duration, this signal will be received by the absorber at $t = 0$. If the emitter sends a second signal at the instant $t = T_e$, let be T_a the instant at which the absorber receives it. When the emitter sends the second signal its position is given by $\vec{r}_e = \vec{v}_e T_e$; when the absorber receives the second signal, its position is given by $\vec{r}_a = \vec{v}_a T_a$. If V is the velocity of the signal, we have:

$$\vec{r}_a - \vec{r}_e = \vec{v}_a T_a - \vec{v}_e T_e = \vec{V}(T_a - T_e) \quad (24)$$

i.e.

$$\frac{T_a}{T_e}(\vec{V} - \vec{v}_a) = \vec{V} - \vec{v}_e \quad (25)$$

and, by taking the scalar product of both members of this equation with \vec{V} , dividing by V^2 and rearranging the terms:

$$\frac{T_a}{T_e} = \frac{1 - (v_e/V) \cos(\vec{V}, \vec{v}_e)}{1 - (v_a/V) \cos(\vec{V}, \vec{v}_a)} \quad (26)$$

or, in terms of frequencies:

$$\frac{f_a}{f_e} = \frac{1 - (v_a/V) \cos(\vec{V}, \vec{v}_a)}{1 - (v_e/V) \cos(\vec{V}, \vec{v}_e)}. \quad (27)$$

If we want to express equation (26) in terms of the proper periods τ_e, τ_a , i.e. of the periods measured in the reference frames co-moving with the emitter and absorber, we must recall that:

$$T_a = \frac{\tau_a}{\sqrt{1 - v_a^2/c^2}}; \quad T_e = \frac{\tau_e}{\sqrt{1 - v_e^2/c^2}}. \quad (28)$$

Therefore, for the proper periods:

$$\frac{\tau_a}{\tau_e} = \frac{1 - (v_e/V) \cos(\vec{V}, \vec{v}_e)}{1 - (v_a/V) \cos(\vec{V}, \vec{v}_a)} \sqrt{\frac{1 - v_a^2/c^2}{1 - v_e^2/c^2}} \quad (29)$$

and, for the proper frequencies (ν_e, ν_a):

$$\frac{\nu_a}{\nu_e} = \frac{1 - (v_a/V) \cos(\vec{V}, \vec{v}_a)}{1 - (v_e/V) \cos(\vec{V}, \vec{v}_e)} \sqrt{\frac{1 - v_e^2/c^2}{1 - v_a^2/c^2}}. \quad (30)$$

In the case of acoustic signals, V is the speed of sound. In the air, $V \approx 340 \text{ ms}^{-1}$ and since v_e and v_a are much smaller than c , the relativistic factors in equations (29), (30) can be neglected and the classical formula (27) is retrieved. Notice also that the time dilation factors in (30) cancel out when $\nu_a = \nu_e$, i.e. when the magnitude of the velocity vectors of emitter and absorber are equal.

For the relativity principle, equations (27) and (30) are valid in every inertial frame; in particular, in the inertial frame of the absorber. As an example, let us consider the classical formula for acoustic signals (27) in the simple case in which emitter and absorber are approaching (in the laboratory reference frame) along the direction of propagation of the signal. In the laboratory reference frame, we have:

$$\frac{f_a}{f_e} = \frac{1 + v_a/V}{1 - v_e/V}. \quad (31)$$

Instead, in the absorber reference frame, equation (27) assumes the form:

$$\frac{f_a}{f_e} = \frac{1}{1 - v_e'/V'}, \quad (32)$$

where $v_e' = v_a + v_e$ and $V' = V + v_a$ are the velocity of the emitter and of the signal in the absorber's reference frame. By substituting these values in (32) we get immediately equation (31): since in newtonian kinematics there is no time dilation, the frequencies have the same value in every inertial reference frame.

In the case of light *in vacuum*, we must substitute V with c in equations (29), (30). In the reference frame of the absorber equation (30) assumes the form:

$$\nu_a = \nu_e \frac{\sqrt{1 - v_e^2/c^2}}{1 - (v_e/c) \cos(\vec{c}, \vec{v}_e)} \quad (33)$$

because in the absorber's reference frame $v_a = 0$; now \vec{v}_e is the velocity of the emitter with respect to the absorber. Equation (33) coincides with that of the luminous Doppler effect in vacuum (18).

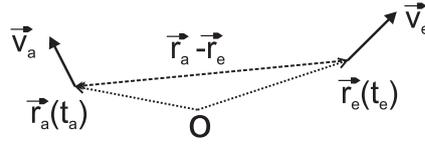


Figure 10. General case. The emitter sends the first signal at the instant t_e when it is at the point \vec{r}_e ; the absorber receives this signal at the instant t_a when it is at the point \vec{r}_a . See the text.

The above results have been derived on the assumption that the emitter emits signals of ideally null duration. More physically, this means that the duration of the signal is much smaller than the time interval between two consecutive signals, i.e. of the period. Therefore, the above derivations are valid also if the signal is constituted by a wave (acoustic or luminous): in this case the signal may be identified with the minimum amplitude (zero) of the wave and its duration by the uncertainty with which the absorber measures the instant at which it measures zero. It goes without saying that, in this case, the interval between two consecutive zeros is equal to half a period of the wave.

Now, we shall show that, as stated above, equations (27) and (30), derived by using a particular initial configuration of the system composed by the emitter and absorber, coincide with those obtainable in the most general case (figure 10).

If the emitter emits the first signal at the instant t_e when it is at the point \vec{r}_e and the absorber receives this signal at the instant t_a when it is at the point \vec{r}_a , we have:

$$\vec{r}_a - \vec{r}_e = \vec{V}(t_a - t_e). \quad (34)$$

If the emitter sends the second signal at the instant $t_e + \Delta t_e$, where Δt_e is a finite increment, the absorber will receive this signal at the instant $t_a + \Delta t_a$. Since the emitter and the absorber move with constant velocity, we may differentiate equation (34) with respect t_e by using finite increments instead of infinitesimal ones. Then, by differentiating and rearranging the terms:

$$\frac{\Delta t_a}{\Delta t_e} (\vec{V} - \vec{v}_a) = \vec{V} - \vec{v}_e. \quad (35)$$

This equation coincides with equation (25), because now Δt_e and Δt_a are the two periods. Therefore, by proceeding as done above from equation (25) onwards, we get again equations (27) and (30).

7.4. The Doppler effect for photons

As shown in section 7.1, in the 19th century, the luminous Doppler effect has been treated within the wave description of light. In the first decade of the 20th century, Stark, as reported in footnote 3, connected somehow the Doppler effect to the energy $h\nu$ of a light quantum. However, the lack of the idea of quantised energy levels in atoms/molecules made it impossible to develop a sound quantitative treatment of the Doppler effect in terms of light quanta. As we shall see below, this became possible in the 1920s, after the incorporation of the idea of quantised energy levels into the background knowledge.

7.4.1. Conservation of linear momentum. After the re-attribution of a linear momentum $h\nu/c$ to a light quantum (Einstein 1916), we must wait until 1921 for its first application. This welcome novelty was due to Robert Emden with a paper entitled ‘On light quanta’. The opening of the paper is a clear exposition of Emden’s programme:

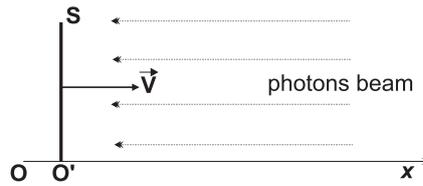


Figure 11. The completely absorbing surface S is moving with velocity V along the positive direction of the x axis in the laboratory reference frame O . A beam of photons with energy $h\nu$ is travelling along the negative direction of the x axis.

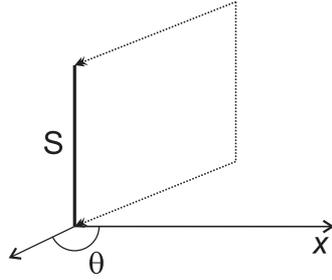


Figure 12. Now the photons beam forms an angle θ with the x axis. The surface S , in one second, absorbs all the photons contained in a volume $S \times (c|\cos \theta| + V)$.

Optical phenomena cannot be reduced to a basic view any more: undulatory theory, once almighty, fails in various fields that can be treated completely and simply with the hypothesis of light quanta. However, within undulatory theory, light quanta are considered, so to speak, as a transitional hypothesis bound, sooner or later, to disappear. The disagreement that results from the contraposition of the two views, can usually be avoided by using coherently the theory that allows the most simple treatment of the phenomena under study. Therefore, in the following, the laws of thermal radiation will be treated on the basis of light quanta without any reminiscence of undulatory theory: this one, in the following, must not exist [14, p 513].

Emden failed to obtain the black-body radiation law by developing a statistics of light quanta enclosed in a cavity at thermal equilibrium. However, his paper contains a sound application of $h\nu/c$ for deriving the formula of the Doppler effect at the first order of v/c . Since Emden treatment is conceptually interesting and has a didactic value, we shall develop Emden's idea in full, firstly by using special relativity and, then, by considering the more general case.

In the laboratory reference frame O , we have (figure 11):

$$F_x = \frac{dP}{dt} = -n \frac{h\nu}{c} (c + V)S, \quad (36)$$

where P is the linear momentum of the surface and n the density of photons. In the reference frame O' co-moving with the surface S we have instead:

$$F'_x = \frac{dP'}{dt'} = -n' \left(\frac{h\nu'}{c} \right) c S' \quad (37)$$

Since $F_x = F'_x$, $S = S'$ and $n' = n\sqrt{1 - V^2/c^2}$ we get immediately:

$$\nu = \nu' \frac{\sqrt{1 - V^2/c^2}}{1 + V/c}. \quad (38)$$

If the photons beam propagates along the positive direction of the x axis, the final formula is given again by equation (38) but with $(-V/c)$ in the denominator.

Let us now consider the more general case in which the incoming beam of photons forms an angle θ with the x axis (figure 12).

In this case, we have in the laboratory reference frame:

$$F_x = \frac{dP_x}{dt} = -n \frac{h\nu}{c} S (c|\cos\theta| + V) = n \frac{h\nu}{c} S (c \cos\theta - V) \quad (39)$$

because in a unit time the surface S absorbs all the photons contained in a volume $S \times (c|\cos\theta| + V)$ (figure 12). Instead, in the frame co-moving with the surface:

$$F'_x = \frac{dP'}{dt'} = n' \frac{h\nu'}{c} (cS') \cos\theta'. \quad (40)$$

Since $F_x = F'_x$, $S = S'$ and (aberration equation)

$$\cos\theta' = \frac{\cos\theta - B}{1 - B \cos\theta} \quad (41)$$

we get:

$$\nu = \nu' \frac{\sqrt{1 - V^2/c^2}}{1 - (V/c) \cos\theta} \quad (42)$$

which is the general formula for the Doppler effect derived by considering light as composed by waves.

If instead of a photons beam we are dealing with a monochromatic, plane electromagnetic wave, we can substitute in the above equations $n(h\nu/c)$ with $\langle u \rangle/c$. Therefore, from equations (39), (40) and from the aberration equation (41) we get:

$$\langle u \rangle = \frac{\langle u' \rangle}{1 - B \cos\theta}. \quad (43)$$

Hence, the average energy contained in a finite volume V is given by:

$$\langle U \rangle = \langle u \rangle V = \frac{\langle u' \rangle}{1 - B \cos\theta} V' \sqrt{1 - V^2/c^2} = \langle U' \rangle \frac{\sqrt{1 - V^2/c^2}}{1 - B \cos\theta} \quad (44)$$

thus showing that the energy of an electromagnetic wave contained in a finite volume transforms, in passing from one inertial frame to another, like the frequency of the wave (equation (42)). This is another way of getting Einstein's result (20).

These derivations of the formulae for the Doppler effect of light are, perhaps, the most simple ones. In particular, this is true for the simple case in which the photon beam propagates along the same direction (or in the opposite direction) of the motion of the absorbing surface. Therefore, they seem particularly suitable for pedagogical purposes. It may be worth adding that, owing to how we have chosen the axis of the two inertial frames, the transformation equation for the x component of the force and that for the surface S are the same in both relativistic and newtonian treatment. The crucial passages in which the

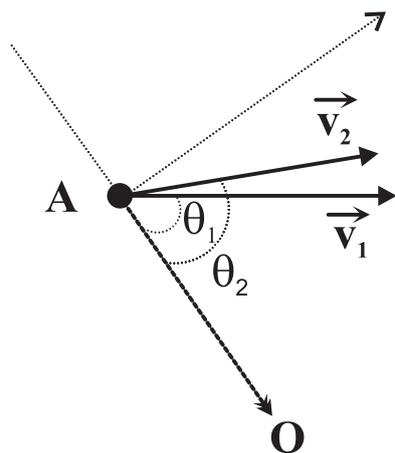


Figure 13. Emission of a photon by the atom A in motion. The photon is emitted along the direction $A \rightarrow O$. The subscript 1 denotes the quantities before the emission; the subscript 2 the quantities after the emission. The phenomenon, which takes place in the plane defined by \vec{v}_1 and AO , is described in the inertial laboratory frame.

relativistic treatment introduces substantial differences are those relative to the transformation of the density of photons n and the aberration equation.

7.4.2. Energy and linear momentum conservation. In 1922, Schrödinger published a paper on the ‘Doppler principle and Bohr’s frequency condition’ [3]. The paper contains much more physics than that announced by the title: it is a treatment of the emission of a photon by a flying atom, described on the basis of the relativistic conservation of energy and linear momentum. This paper has been overlooked by Schrödinger’s contemporaries and, also nowadays, it is not so popular. As evidence of this neglect, we can recall that Juancey’s [59] and Davisson’s [60] subsequent papers on the same subject are a kind of involuntary, concise presentation of the unquoted Schrödinger’s one. Similarly, Fermi, in the Introduction of his ‘Quantum theory of radiation’ develops the same calculation carried out by Schrödinger, as a first approximation, by using classical mechanics [61, p 105]: but Fermi does not cite Schrödinger. Schrödinger’s paper has been recovered by Redžić [62]; one of the present authors has applied its physics to the description of the emission of electromagnetic radiation by atoms or nuclei in inertial flight at relativistic speed [63], to the emission/absorption of photons without recoil (Mössbauer effect) on rotating devices [64] and to the laser cooling of atoms [65].

Let us consider the emission of a photon by an atom in flight in the inertial laboratory frame (figure 13).

The energy conservation imposes that:

$$E_{\text{ph}} = \gamma_1 E_1 - \gamma_2 E_2 \quad (45)$$

while the linear momentum conservation implies that:

$$\gamma_1 \frac{E_1}{c^2} v_1 \cos \theta_1 = \gamma_2 \frac{E_2}{c^2} v_2 \cos \theta_2 + \frac{E_{\text{ph}}}{c}, \quad (46)$$

$$\gamma_1 \frac{E_1}{c^2} v_1 \sin \theta_1 = \gamma_2 \frac{E_2}{c^2} v_2 \sin \theta_2. \quad (47)$$

E_{ph} is the energy of the emitted photon; E_1 and E_2 are the rest energies of the atom before and after the emission; γ_1 , γ_2 are the relativistic factors before and after the emission; v_1 and v_2 the atom's velocities before and after the emission; θ_1 and θ_2 the angles between \vec{v}_1 and \vec{v}_2 and the direction of the emitted photon. Notice that $E_1 - E_2 = \Delta E$, where ΔE is the energy difference between the two levels of the atomic transition: ΔE is a relativistic invariant, since it is given by the difference of two rest energies. Schrödinger did not introduce explicitly the transition energy ΔE ; furthermore, he treated only the case of emission. We shall not develop the calculations here since they can be found, in addition to the original paper, in [62, 65]. Instead, we shall focus on the physical implications of the results. The energy of the emitted photon turns out to be:

$$\begin{aligned} E_{\text{ph}}^{\text{emi}} &= \Delta E \left(1 - \frac{\Delta E}{2E_1^{\text{emi}}} \right) \frac{\sqrt{1 - B_1^2}}{1 - B_1 \cos \theta_1} \\ &= \Delta E \left(1 + \frac{\Delta E}{2E_2^{\text{emi}}} \right) \frac{\sqrt{1 - B_2^2}}{1 - B_2 \cos \theta_2}. \end{aligned} \quad (48)$$

In the case of absorption of a photon, after having adequately rewritten the conservation laws, we obtain:

$$\begin{aligned} E_{\text{ph}}^{\text{abs}} &= \Delta E \left(1 + \frac{\Delta E}{2E_1^{\text{abs}}} \right) \frac{\sqrt{1 - B_1^2}}{1 - B_1 \cos \theta_1} \\ &= \Delta E \left(1 - \frac{\Delta E}{2E_2^{\text{abs}}} \right) \frac{\sqrt{1 - B_2^2}}{1 - B_2 \cos \theta_2}. \end{aligned} \quad (49)$$

In the above formulae, the energy of the emitted/absorbed photon is given in terms of the atom's parameters before (subscript 1) and after the emission/absorption (subscript 2). Generally, we are interested in the parameters before emission/absorption. However, the full equations (49) allow to derive, for instance, the atom's velocity after the absorption from the knowledge of the parameters before absorption. For example, these equations can be used while treating the laser cooling of ions flying at relativistic speeds in storage rings [65, p 17].

Care must be taken in handling the terms $\Delta E/2E_1$ and $\Delta E/2E_2$. Let us consider, for example, the case of emission. In this case, we have:

$$\frac{\Delta E}{2E_1^{\text{emi}}} = \frac{\Delta E}{2(Mc^2 + \Delta E)}, \quad (50)$$

$$\frac{\Delta E}{2E_2^{\text{emi}}} = \frac{\Delta E}{2Mc^2}. \quad (51)$$

If $\Delta E \ll Mc^2$, we can put $\Delta E/2E_1^{\text{emi}} = \Delta E/2Mc^2$: generally, this approximation is valid for transitions between two atomic levels, but not for transitions between two nuclear levels. The energy $E_R = \Delta E^2/2Mc^2$ is called recoil energy. Exactly, it is the kinetic energy of an atom after the absorption when it is at rest before absorption; and, within the validity of the inequality $\Delta E \ll Mc^2$, it is also the kinetic energy of an atom after the emission when it is at rest before emission. Within the validity of this approximation, the energy of the emitted photon (48) can be written as:

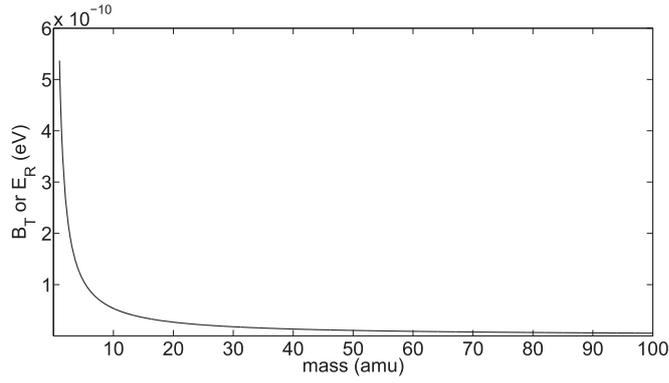


Figure 14. The dimensionless parameter $B_T = \Delta E/2Mc^2 = E_R/\Delta E$ is plotted as a function of the atom's mass in atomic unit for a transition energy $\Delta E = 1\text{eV}$. The values plotted on the y axis are also those of the recoil energy expressed in eV.

$$E_{\text{ph}}^{\text{emi}} = (\Delta E - E_R) \frac{\sqrt{1 - B_1^2}}{1 - B_1 \cos \theta_1} = \Delta E (1 - B_T) \frac{\sqrt{1 - B_1^2}}{1 - B_1 \cos \theta_1}, \quad (52)$$

where $B_T = \Delta E/2Mc^2 = E_R/\Delta E$ is a dimensionless, small, parameter: see figure 14.

In spite of its smallness, B_T plays a basic role in many physical phenomena since it controls the atom's recoil energy and allows a comparison at a glance with the values of B_1 (or B_2) (the atom's speed in units of c) when both enter the same formula. As shown in [65], this is particularly useful in dealing with laser cooling of atoms. As far as the recoil energy, its value is of fundamental importance with respect to the natural linewidth of the atomic or nuclear transition. If the recoil energy is smaller than the linewidth, a photon emitted by an atom at rest can be absorbed by another one at rest: this usually happens in atomic transitions. If the recoil energy is much larger than the natural linewidth, the resonance absorption of an emitted photon is impossible: this may happen in the case of nuclear transitions.

For the energy of the absorbed photon we have exactly:

$$E_{\text{ph}}^{\text{abs}} = (\Delta E + E_R) \frac{\sqrt{1 - B_1^2}}{1 - B_1 \cos \theta_1} = \Delta E (1 + B_T) \frac{\sqrt{1 - B_1^2}}{1 - B_1 \cos \theta_1}. \quad (53)$$

In order to illustrate a role played by B_T , let us consider the emission of a photon along the direction of the atom's velocity vector. The energy of the emitted photon is given by:

$$E_{\text{ph}}^{\text{emi}} = \Delta E (1 - B_T) \sqrt{\frac{1 + B_1}{1 - B_1}}. \quad (54)$$

Putting $E_{\text{ph}}^{\text{emi}} = \Delta E$, equation (54) yields for B_1 :

$$B_1 = \frac{2B_T - B_T^2}{2 - 2B_T + B_T^2}. \quad (55)$$

Taking into account the order of magnitude of B_T (figure 14), we can neglect, with a good approximation, the term $-B_T^2$ in the numerator and the terms $-2B_T$ and B_T^2 in the denominator. Then: $B_1 = B_T$. It follows that:

$$E_{\text{ph}}^{\text{emi}} < \Delta E \quad \text{for } B_1 < B_T \quad (56)$$

$$E_{\text{ph}}^{\text{emi}} = \Delta E \quad \text{for } B_1 = B_T \quad (57)$$

$$E_{\text{ph}}^{\text{emi}} > \Delta E \quad \text{for } B_1 > B_T \quad (58)$$

B_T operates as a threshold parameter. In particular, it is not true that the energy of the emitted photon along the direction of motion of the atom is always larger than the transition energy ΔE . Instead the energy of a photon emitted in the opposite direction of the atom's velocity vector is:

$$E_{\text{ph}}^{\text{emi}} = \Delta E (1 - B_T) \sqrt{\frac{1 - B_1}{1 + B_1}} \quad (59)$$

which is always smaller than ΔE .

In the case of absorption, we have, for a photon absorbed in a head-on collision:

$$E_{\text{ph}}^{\text{abs}} = \Delta E (1 + B_T) \sqrt{\frac{1 - B_1}{1 + B_1}}. \quad (60)$$

Therefore, proceeding as in the case of the emitted photon, we find that:

$$E_{\text{ph}}^{\text{abs}} > \Delta E \quad \text{for } B_1 < B_T, \quad (61)$$

$$E_{\text{ph}}^{\text{abs}} = \Delta E \quad \text{for } B_1 = B_T, \quad (62)$$

$$E_{\text{ph}}^{\text{abs}} < \Delta E \quad \text{for } B_1 > B_T. \quad (63)$$

Also in this case, B_T operates as a threshold parameter. When the photon to be absorbed is flying in the same direction of the atom, we have:

$$E_{\text{ph}}^{\text{abs}} = \Delta E (1 + B_T) \sqrt{\frac{1 + B_1}{1 - B_1}} \quad (64)$$

which is always larger than ΔE .

The variation of the atom's kinetic energy due to the absorption of a photon is given by:

$$\Delta E_K^{\text{abs}} = (\gamma_2 E_2^{\text{abs}} - E_2^{\text{abs}}) - (\gamma_1 E_1^{\text{abs}} - E_1^{\text{abs}}) = E_{\text{ph}}^{\text{abs}} - \Delta E \quad (65)$$

which, when the atom is at rest before absorption, yields $\Delta E_K^{\text{abs}} = E_R$. In the case of the absorption of a photon in a head-on collision, we have (taking into account equations (61)–(63)):

$$\Delta E_K^{\text{abs}} > 0 \quad \text{for } B_1 < B_T, \quad (66)$$

$$\Delta E_K^{\text{abs}} = 0 \quad \text{for } B_1 = B_T, \quad (67)$$

$$\Delta E_K^{\text{abs}} < 0 \quad \text{for } B_1 > B_T. \quad (68)$$

The case of $B_1 > B_T$ is the basic mechanism exploited in laser cooling of two levels atoms [65]. However, when $B_1 < B_T$, the variation of the atom's kinetic energy is positive. If the photon to be absorbed is flying in the same direction of the atom, the variation of the atom kinetic energy is always positive.

In the case of emission of a photon, the variation of the atom's kinetic energy is given by:

$$\Delta E_K^{\text{emi}} = (\gamma_2 E_2^{\text{emi}} - E_2^{\text{emi}}) - (\gamma_1 E_1^{\text{emi}} - E_1^{\text{emi}}) = \Delta E - E_{\text{ph}}^{\text{emi}}. \quad (69)$$

Therefore, when the photon is emitted along the direction of the atom's motion (taking into account equations (56) and (57)):

$$\Delta E_K^{\text{emi}} > 0 \quad \text{for } B_1 < B_T, \quad (70)$$

$$\Delta E_K^{\text{emi}} = 0 \quad \text{for } B_1 = B_T, \quad (71)$$

$$\Delta E_K^{\text{emi}} < 0 \quad \text{for } B_1 > B_T. \quad (72)$$

Finally, the variation of the kinetic energy of the atom due to the emission of a photon in the opposite direction of the atom velocity vector is always positive.

As shown in [65], these results, generalised to the case of emission/absorption of a photon along an arbitrary direction (with respect to the line of motion of the atom) are the building blocks of the theoretical treatment of laser cooling of two levels atoms.

Finally, it is interesting to see that the energy of an emitted/absorbed photon can be derived in another way. For example, in the case of emission, we first derive, through the conservation equations, the energy of the emitted photon when the atom is at rest before emission. For brevity, we omit the derivation; also because the result may be simply obtained by putting $B_1 = 0$ in equation (48). The result is $E_{\text{ph}}^{\text{emi}^0} = \Delta E(1 - B_T)$: a relativistic invariant. When the atom is in flight with velocity \vec{v}_1 , we consider, in addition to the laboratory reference frame O , a reference frame O' co-moving with the atom before emission: we take the $x \equiv x'$ axis directed along \vec{v}_1 . Then, in O' the photon will be emitted with energy $\Delta E(1 - B_T)$, along, say the direction forming an angle θ' with the axis $x' \equiv x$. If we treat the photon as a relativistic particle with zero mass, we can consider its energy-momentum four-vector $(E_{\text{ph}}^{\text{emi}}, \vec{p})$ with $p = E_{\text{ph}}^{\text{emi}}/c$ and write the transformation equation for its component $E_{\text{ph}}^{\text{emi}}$.

$$E_{\text{ph}}^{\text{emi}'} = \Delta E(1 - B_T) = \gamma E_{\text{ph}}^{\text{emi}}(1 - B_1 \cos \theta) \quad (73)$$

which, solved for $E_{\text{ph}}^{\text{emi}}$ yields immediately equation (52). Equation (52) can be rewritten as:

$$E_{\text{ph}}^{\text{emi}} = E_{\text{ph}}^{\text{emi}'} \frac{\sqrt{1 - B_1^2}}{1 - B_1 \cos \theta_1}, \quad (74)$$

where $E_{\text{ph}}^{\text{emi}'} = \Delta E(1 - B_T)$. If we put $E_{\text{ph}}^{\text{emi}} = h\nu$ and $E_{\text{ph}}^{\text{emi}'} = h\nu'$, equation (74) becomes the formula for the luminous Doppler effect in vacuum written in terms of waves.

The extended Schrödinger's treatment of the emission/absorption of a light quantum by an atom/nucleus is an application of the original Einstein idea that the emission of light on a microscopic scale is a directional process. It shows that the Doppler effect is a direct consequence of energy and linear momentum conservation laws. The connection with the wave description is assured by the fundamental relation $E_{\text{ph}} = h\nu$.

8. What is light?

So far, we have spoken of 'descriptions' of light, undulatory or corpuscular. To a description of a phenomenon, or of a series of phenomena, we ask to yield predictions that are in agreement with experiment and, possibly, to cover all the known features of the phenomenon(a). Usually, when a description satisfies, at least in principle, these conditions, we label it as a theory. Nowadays, physical theories are mathematical theories. In the past, it was not so. For instance, Michael Faraday in his *Experimental Researches* developed a detailed theory of electromagnetic induction without writing any formula.

Theories use theoretical terms (theoretical entities) like, for instance, electron, atom, wave or photon. The equations of a theory contain physical quantities that describe properties of, or relations between, theoretical terms. For instance, the equations of the wave theory of light may contain, as physical quantities, wavelength, frequency or intensity. On the other hand, the equations of the photon theory of light may contain physical quantities such as energy or momenta of the photons or probabilities. Examples of physical quantities that describe relations between theoretical entities are those of velocity and force.

Generally speaking, physical quantities can be measured by procedures that are suggested by the theory itself and/or by the acquired knowledge. The fact that a physical quantity is measured does not imply that its theoretical entity exists in the World. Moreover, the fact that the predictions of a theory are in agreement with experiment, does not imply that, in the World, things go exactly as described by the theory. Maxwell, in his contribution to the IX edition of the *Encyclopædia Britannica* entitled ‘Ether’, showed how to measure the rigidity of the ether, starting from the measurement, on the Earth, of the energy coming from the Sun [66]. The existence of the ether was plausible in those times; it is not nowadays. Therefore, we can say that Maxwell measured a property of something that does not exist. Another, more intriguing and subtle example can be taken from solid state physics. If in the valence band of a semiconductor all the N electronic states are occupied except one, we say that there is a ‘hole’ in the valence band. A hole has, as physical properties, the electric charge, the effective mass and the electrical mobility. All of them, are measurable quantities and are currently measured. However, the same theory which has invented the ‘hole’ explains that it describes the properties of the $N - 1$ electrons that occupy all the states of the valence band except one. Therefore, the ‘hole’ does not exist in the World, at least in the sense that we attribute to the statement of the existence of the electron [67]. On this issue, William Shockley, one of the three fathers of the transistor, held an operational stand: ‘From the theoretical viewpoint, the hole is an abstraction from a much more complex situation and the achieving of this abstraction in a logical way appears inevitably to involve rather detailed quantum-mechanical considerations. From the experimental viewpoint, in contrast, the existence of holes and electrons as positive and negative carriers of current can be inferred directly by the experimental techniques of transistor electronics so that holes and electrons have acquired an operational reality in Bridgman’s sense of the word’ [68, p IX]. The cases of the ether and that of the holes constitute clear examples of a methodological principle: we can measure the physical quantities associated with a theoretical entity without having any reasonable basis for attributing a real existence to that entity. On a more general level, it must be added that the existence in the World of a theoretical entity is never an explicit postulate of a theory and, anyway, it is never used in the mathematical or logical deductions of the theory. In conclusion, it seems that any assertion of existence (ontological statement) can be made only *a posteriori*, must be compatible with the entire acquired knowledge and is only plausible. When an ontological statement is made it asserts the plausible existence of an object in the world¹⁰.

An ontological assertion about the plausible existence of a theoretical entity may last much more than the theory that contains it. For instance, the theory of the electron or of the hydrogen atom has profoundly changed from Lorentz’s theory of the electron or from Bohr’s

¹⁰ In [69, p 879], one of the present authors proposed a possible example of a statement of existence of the electron: ‘In the World there is a *quid* which has properties that correspond to the properties attributed by our theory to the ‘electron’ and this *quid* behaves in accordance with the laws of our theory and with properties that are described by the measured values of the physical quantities that our theory attributes to the ‘electron’. We can convene that the statement the electron exists in the World is a shorthand of the previous one.’ Anyway, any statement of this kind must be compatible with the entire acquired knowledge.

theory of the hydrogen atom, respectively. The plausible existence of the electron or of the hydrogen atom not only has survived their primitive theories, but the plausibility of their existence has strengthened in time.

Since ontological statements about the existence of theoretical entities are only plausible, one may wonder which is the utility, if any, of statements of this kind. The answer is that ontological statements are the building blocks of Images of the World that guide us in our daily life, the experimenter in his laboratory and the theorist in the elaboration of his theories.

If the question of this section is what is light in the World, we can give only a *plausible* answer, based on our acquired theoretical and experimental knowledge and compatible with it. Then, we can say that light is constituted by energy quanta, called photons, which are endowed with linear and intrinsic angular momentum. These photons cannot be divided by any means and their emission or absorption by matter (endowed with mass) is regulated by energy and momentum (linear and angular) conservation laws. The detection of a photon occurs on a small area of the detector.

What about the existence in the World of electromagnetic waves? When we are dealing with one photon at a time that interacts with a detector, it is hard to say that an electromagnetic wave is there. When we are dealing with statistically significant numbers of photons used all together we might say plausibly that an electromagnetic wave is there. However, in the double slit experiment with one photon at a time when the total number of photons used is high enough, we cannot say that a wave is there, but only that the wave description yields the correct predictions. It seems that ontological statements about the existence of electromagnetic waves are not so plain as in the case of photons. Anyway, we must keep in mind that the Images of the World are only plausible and, often, incoherently assembled. When used as a heuristic guide, they sometimes lead towards rewarding paths some other into dead alleys.

9. Conclusions

We have shown that interference phenomena and the Doppler effect for light in vacuum can be described in terms of waves or in terms of photons. However, the two descriptions are not equivalent. While the photon description deals with a single photon, the wave one deals with the behaviour of many (enough) photons. In the case of the Doppler effect, the wave description reduces to the transformation equation of the wave frequency between two inertial reference frames. Instead, the quanta description is founded on conservation equations or on conservation equations plus the transformation equation of the photon's energy. The photon description allows to determine, for instance, the energy of the photon emitted when the atom is at rest before emission, energy that is smaller than ΔE (energy difference between the two energy levels of the atomic/nuclear transition), owing to the atom's recoil energy.

In the case of interference, the photon description explains why the images are built up spot by spot. The two descriptions have the same mathematical structure and the wave description is valid only when the number of light quanta used (one at a time or all together) are large enough to overshadow the localised interaction between quanta and detector.

Generally speaking, the predictions of the wave theory of light are in agreement with experiment when the number of photons involved is large enough, regardless if they are used all together or one at a time.

'Quantons', i.e. theoretical entities (like photons, electrons, atoms, nuclei...) used by quantum theoretical physics cannot be adequately referred to in terms of waves *or* in terms of particles. This statement is grounded in the fact that, for instance, the electron has among its

physical properties not only classical items like mass and electric charge but also de Broglie's wavelength. Then, it is true that the electron is described by properties typical of a particle and of a wave. The concept of 'quanton' allows one to formally supersede the wave-particle duality expressed by saying that, for instance, an electron behaves sometimes as a particle and some other as a wave. In fact, once we have accepted this new concept, we can only say that quantons behave as quantons. However, at the present, we do not know if the concept of 'quanton' is only the product of a historical superposition of different layers (theories and Images of the World) that, in turn, will be superseded by a new insight which should also explain why Maxwell's theory of light is valid when the number of photons is high enough.

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